PIE Learning Pathways

Pathways for Instructionally Embedded Assessment

Pathways for Instructionally Embedded Assessment

Missouri is leading the way to develop assessments that work better for teachers and students! The Missouri Department of Elementary and Secondary Education (DESE) leads the Pathways for Instructionally Embedded (PIE) Assessment project in partnership with the Accessible Teaching, Learning, and Assessment Systems (ATLAS) at the University of Kansas.

As a research and development project, the goal of the PIE project is to:

- construct and evaluate a prototype assessment system built on cognitive models of student learning, known as learning pathways, aligned with grade-level academic expectations.
- deliver instructionally embedded assessments that can provide students the opportunity to demonstrate their knowledge, skills, and understandings **as they learn** while providing teachers with timely information about their students' learning progress.

Learning Pathways

Learning progressions are descriptions of "successively more sophisticated ways of reasoning within a content domain that follow one another as students learn"[1] and are useful tools that can improve assessments using research on student thinking and learning.

PIE learning pathways are based on research and show how students can move from the basic cognitive and early academic skills they currently have towards the knowledge, skills and understanding (KSUs) that they need to master the grade-level standards. The progressions of these skills are shown on maps and include many different and interrelated paths that students may take to get to those targeted skills. The maps help educators to understand how those skills might logically follow one another.

Each learning pathway includes three levels, Levels 1, 2, and 3. These pathway levels are vertical and are critical towards the KSUs that are required to master the skills needed at the content standard. Level 1 and Level 2 skills are the foundational skills. Level 3 skills represent the content standard. There are increases in cognitive complexity of the skills as they move from Level 1 to Level 3. Therefore, the pathways show how these skills vertically progress towards the learning targets at the standard level.

Development of the Learning Pathways

The Missouri DESE selected and prioritized 25 grade 5 mathematics content standards from the Priority Standards for Leveraging Learning in Mathematics for learning pathway development[2]. The 25 prioritized mathematics content standards included (a) Number Sense and Operations in Fractions (NF; 14 standards from two clusters, 5.NF.A and 5.NF.B); (b) Relationships and Algebraic Thinking (RA; six

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[1] Smith, C. L., Wiser, M., Anderson, C. W., & Krajcik, J. (2006). Implications of research on children's learning for standards and assessment: A proposed learning progression for matter and the atomic-molecular theory. *Measurement: Interdisciplinary Research & Perspectives*, *4*(1–2), 1–98. https://doi.org/10.1080/15366367.2006.9678570

[2] Missouri Department of Elementary and Secondary Education (DESE). (2021, April). *Priority standards for leveraging learning in mathematics: Grades K–12*. https://sites.google.com/view/priority-standards-mo-dese/home/mathematics?authuser=0

standards from two clusters, 5.RA.A and 5.RA.C); (c) Geometry and Measurement (GM; four standards from three clusters, 5.GM.A, 5.GM.B, 5.GM.C, and 5.GM.D); and (d) Data and Statistics (DS; one standard from one cluster, 5.DS.A).

The learning pathways were developed using the process below:
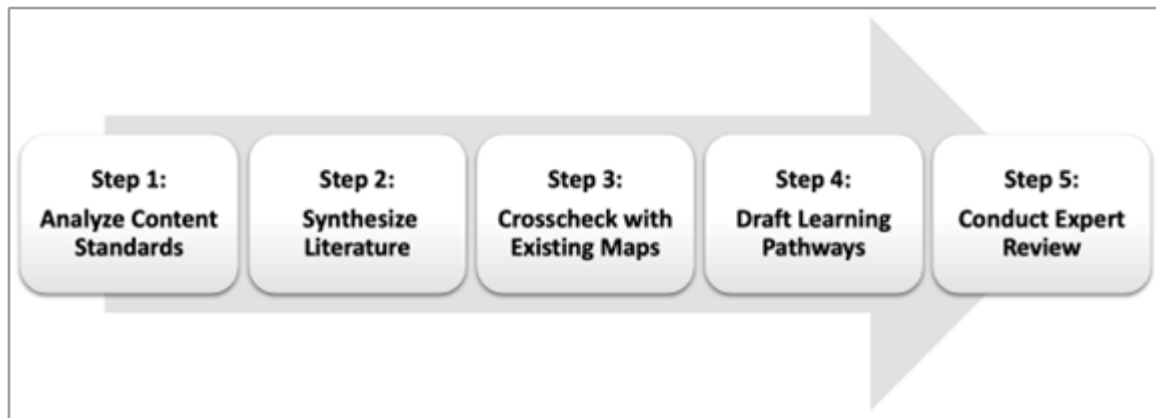
The first four steps were completed by math content experts and researchers at ATLAS. The fifth step, Expert Review, included Missouri educators. The expert review was completed in Columbia, Missouri in June 2023. Educators across the state, representing various districts, participated in the review.

Use of the Learning Pathways
Items on the PIE item field test were designed to measure the KSUs found in the learning pathways. We will use the information collected from the item field test to verify that the items are working as intended. The approved items will be used in a full pilot study of the PIE system (during the 2024-2025 academic year) which includes instructionally embedded assessments based on the learning pathways and assessment results that show which pathway levels students have mastered and not yet mastered.

Cognitive learning models, such as the PIE learning pathways, can help educators pinpoint where students are at in their current learning progress towards grade-level expectations (rather than focus on students' lack of mastery of grade-level skills) and facilitate instructional and learning activities according to students' needs.

Table of Contents

PIE.5.NF.A.1
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.

Learning Target

5.NF.A. Understand the relationship between fractions and decimals (denominators that are factors of 100).
   1. Understand that parts of a whole can be expressed as fractions and/or decimals.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Recognize fraction and decimal notations.

↓

Level 2
Represent whole and fractional parts in a set model.

↓

Level 3
Understand that parts of a whole can be expressed as fractions and/or decimals.

PIE.5.NF.A.1 Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-96
Partition sets

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-99
Partition sets into equal subsets

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

L1 PIE-M-102
Recognize fraction

L1 PIE-M-103
Recognize decimal

PIE-M-104
Explain numerator and denominator

PIE-M-105
Explain unit fraction

PIE-M-106
Explain the decimal point

L2 PIE-M-107
Represent whole in a set model

PIE-M-108
Represent whole on an area model

PIE-M-111
Explain place value for decimals

L2 PIE-M-109
Represent fractional parts in a set model

PIE-M-110
Represent fractional parts on an area model

L3 PIE-M-112
Represent fractional parts with fraction notation

L3 PIE-M-113
Represent fractional parts with decimal notation

PIE-M-114
Recognize equivalent fractions

PIE-M-115
Convert between fractions and decimals

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.NF.A.2
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.

Learning Target

5.NF.A. Understand the relationship between fractions and decimals (denominators that are factors of 100).
   2. Convert decimals to fractions and fractions to decimals.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Explain numerator, denominator, and decimal point.

↓

Level 2
Explain place value for decimals, and recognize equivalent fractions.

↓

Level 3
Convert decimals to fractions and fractions to decimals.

PIE.5.NF.A.2 Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-103
Recognize decimal

PIE-M-105
Explain unit fraction

L1 PIE-M-104
Explain numerator and denominator

L1 PIE-M-106
Explain the decimal point

PIE-M-108
Represent whole on an area model

PIE-M-110
Represent fractional parts on an area model

L2 PIE-M-111
Explain place value for decimals

PIE-M-116
Count by fractional parts

PIE-M-112
Represent fractional parts with fraction notation

PIE-M-113
Represent fractional parts with decimal notation

PIE-M-117
Compare fractions and decimals to benchmarks

L2 PIE-M-114
Recognize equivalent fractions

L3 PIE-M-115
Convert between fractions and decimals

PIE-M-118
Compare fractions and/or decimals up to thousandths using symbols

PIE-M-119
Order fractions and/or decimals up to thousandths using symbols

PIE-M-120
Evaluate results of fraction and/or decimal comparisons

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.A.3

Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
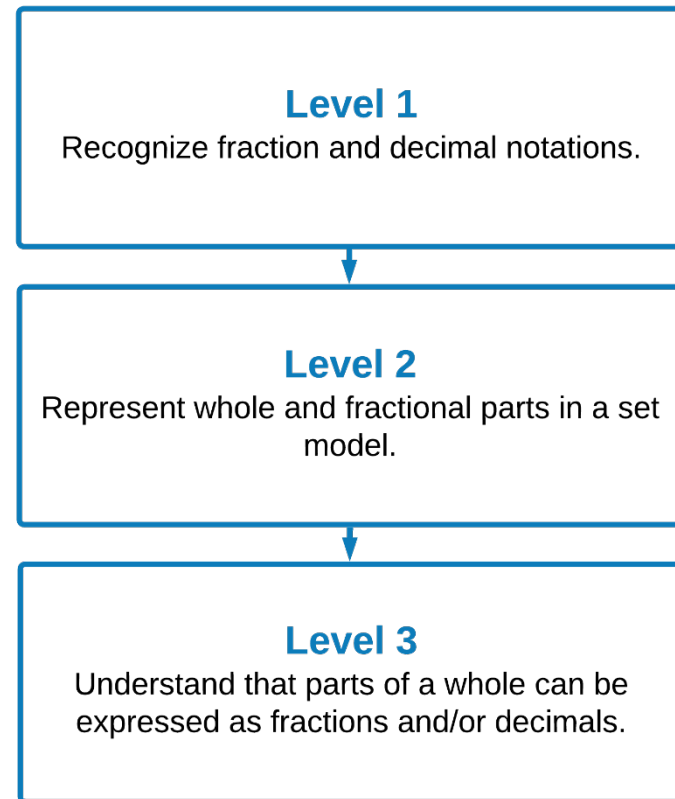
Learning Target

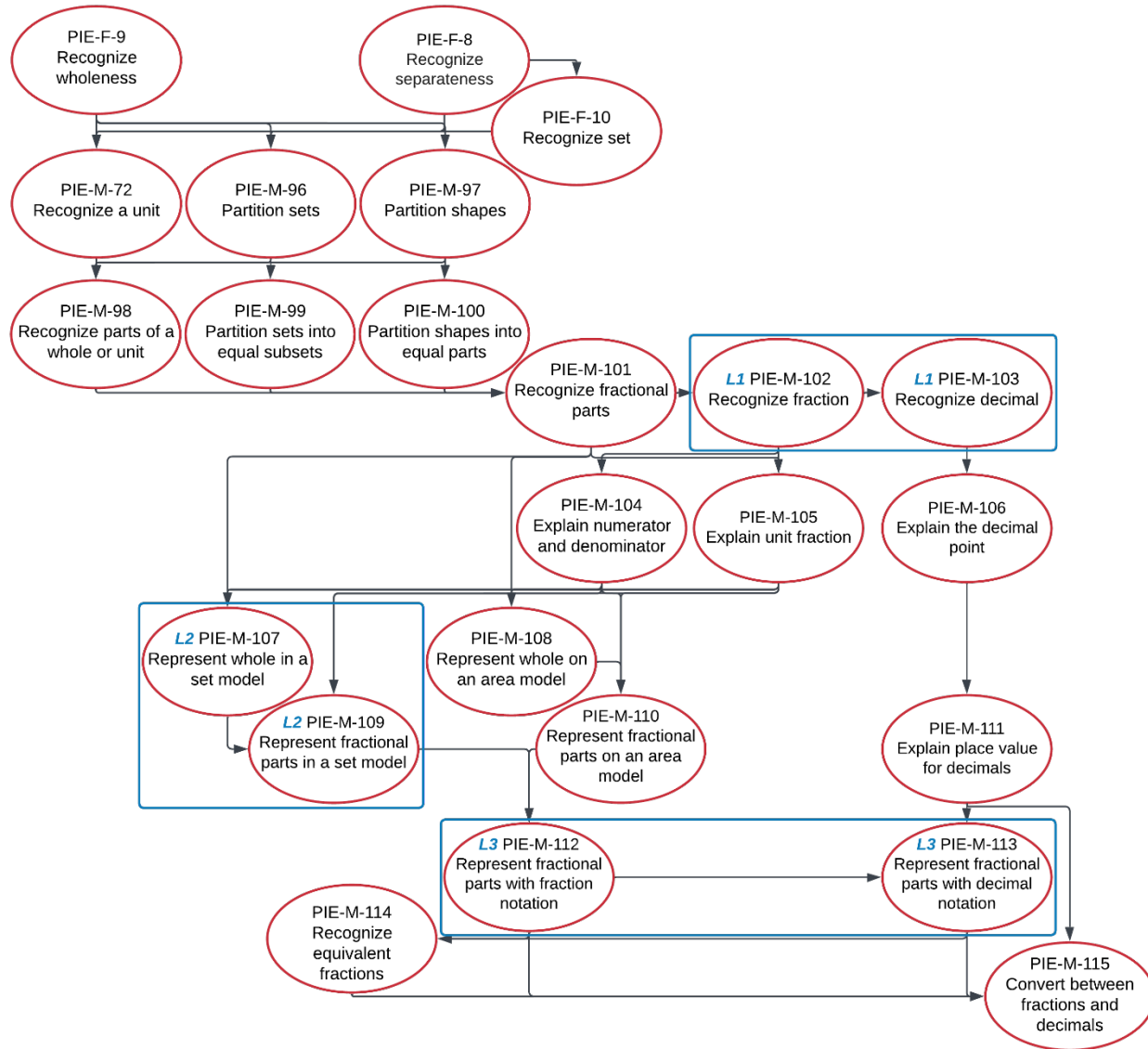**5.NF.A.** Understand the relationship between fractions and decimals (denominators that are factors of 100).
  3. Compare and order fractions and/or decimals to the thousandths place using the symbols >, = or <, and justify the solution.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Compare unit fractions.

↓

Level 2
Compare fractions with like denominators and like decimals.

↓

Level 3
Compare and order fractions and/or decimals up to the thousandths place using symbols (<, >, =) and justify the solution.

PIE.5.NF.A.3 Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-103
Recognize decimal

PIE-M-104
Explain numerator and denominator

L1 PIE-M-121
Compare unit fractions

PIE-M-105
Explain unit fraction

PIE-M-106
Explain the decimal point

PIE-M-122
Represent whole on a length model

PIE-M-123
Represent fractional parts on a length model

PIE-M-124
Compare fractional parts

PIE-M-116
Count by fractional parts

L2 PIE-M-125
Compare fractions with like denominators

L2 PIE-M-126
Compare like decimals

PIE-M-111
Explain place value for decimals

PIE-M-127
Locate fractions and decimals on a number line

PIE-M-117
Compare fractions and decimals to benchmarks

PIE-M-114
Recognize equivalent fractions

PIE-M-115
Convert between fractions and decimals

PIE-M-128
Explain rounding for decimals

L3 PIE-M-118
Compare fractions and/or decimals up to thousandths using symbols

L3 PIE-M-119
Order fractions and/or decimals up to thousandths using symbols

L3 PIE-M-120
Evaluate results of fraction and/or decimal comparisons

PIE-M-129
Estimate results of fraction addition and subtraction

PIE-M-130
Estimate results of decimal addition and subtraction up to thousandths

Map Key

L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.B.4
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
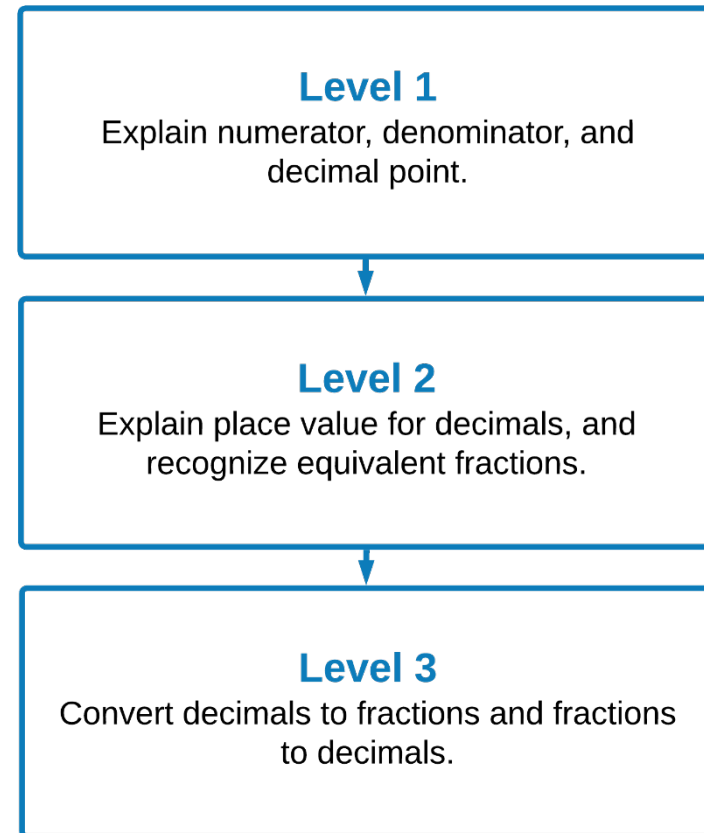
Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.

   4. Estimate results of sums, differences and products with fractions and decimals to the thousandths.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Compare fractional parts.

↓

Level 2
Compare fractions and decimals to benchmarks.

↓

Level 3
Estimate results of sums, differences, and products with fractions and decimals up to thousandths.

PIE.5.NF.B.4 Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-103
Recognize decimal

PIE-M-104
Explain numerator and denominator

PIE-M-105
Explain unit fraction

PIE-M-106
Explain the decimal point

PIE-M-121
Compare unit fractions

PIE-M-122
Represent whole on a length model

PIE-M-123
Represent fractional parts on a length model

PIE-M-111
Explain place value for decimals

L1 PIE-M-124
Compare fractional parts

PIE-M-116
Count by fractional parts

PIE-M-125
Compare fractions with like denominators

PIE-M-126
Compare like decimals

PIE-M-127
Locate fractions and decimals on a number line

L2 PIE-M-117
Compare fractions and decimals to benchmarks

PIE-M-114
Recognize equivalent fractions

PIE-M-128
Explain rounding for decimals

L3 PIE-M-129
Estimate results of fraction addition and subtraction

L3 PIE-M-130
Estimate results of decimal addition and subtraction up to thousandths

L3 PIE-M-132
Estimate results of fraction multiplication

L3 PIE-M-133
Estimate results of decimal multiplication up to thousandths

PIE-M-134
Solve problems involving fraction addition and subtraction with unlike denominators

PIE-M-135
Add and subtract decimals up to thousandths

PIE-M-136
Solve problems involving fraction and whole number multiplication

PIE-M-137
Multiply decimals up to hundredths

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.NF.B.5a
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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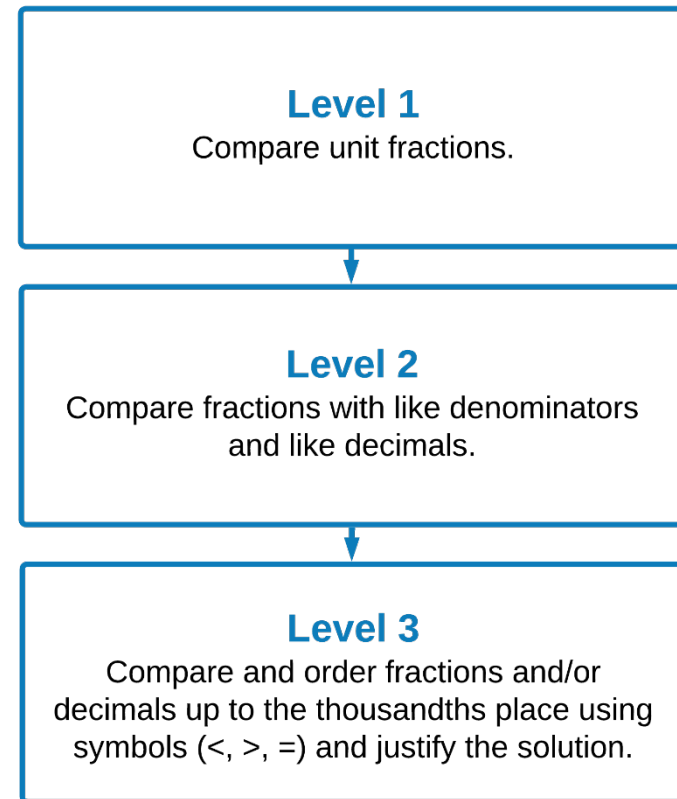
Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.

   5. Justify the reasonableness of a product when multiplying with fractions.

      a) Estimate the size of the product based on the size of the two factors.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Represent fractional parts on a length model.

Level 2
Represent equivalent fractions with length models.

Level 3
Estimate the size of the product of two fractions based on the size of the factors.

PIE.5.NF.B.5a Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator and denominator

PIE-M-105
Explain unit fraction

PIE-M-121
Compare unit fractions

PIE-M-122
Represent whole on a length model

L1 PIE-M-123
Represent fractional parts on a length model

PIE-M-125
Compare fractions with like denominators

PIE-M-116
Count by fractional parts

PIE-M-124
Compare fractional parts

PIE-M-127
Locate fractions and decimals on a number line

PIE-M-117
Compare fractions and decimals to benchmarks

PIE-M-114
Recognize equivalent fractions

L2 PIE-M-138
Represent equivalent fractions with length models

PIE-M-139
Represent fraction multiplication with length models

L3 PIE-M-140
Estimate size of product based on size of fraction factors

PIE-M-141
Explain size of fraction product when a factor is greater than 1

PIE-M-142
Explain size of fraction product when a factor is less than 1

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.B.5b

Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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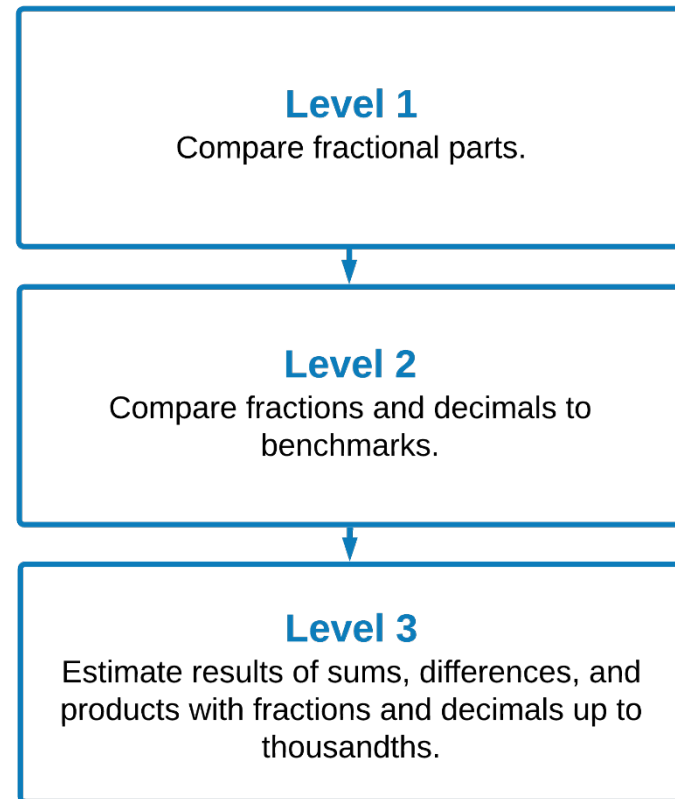
Learning Target

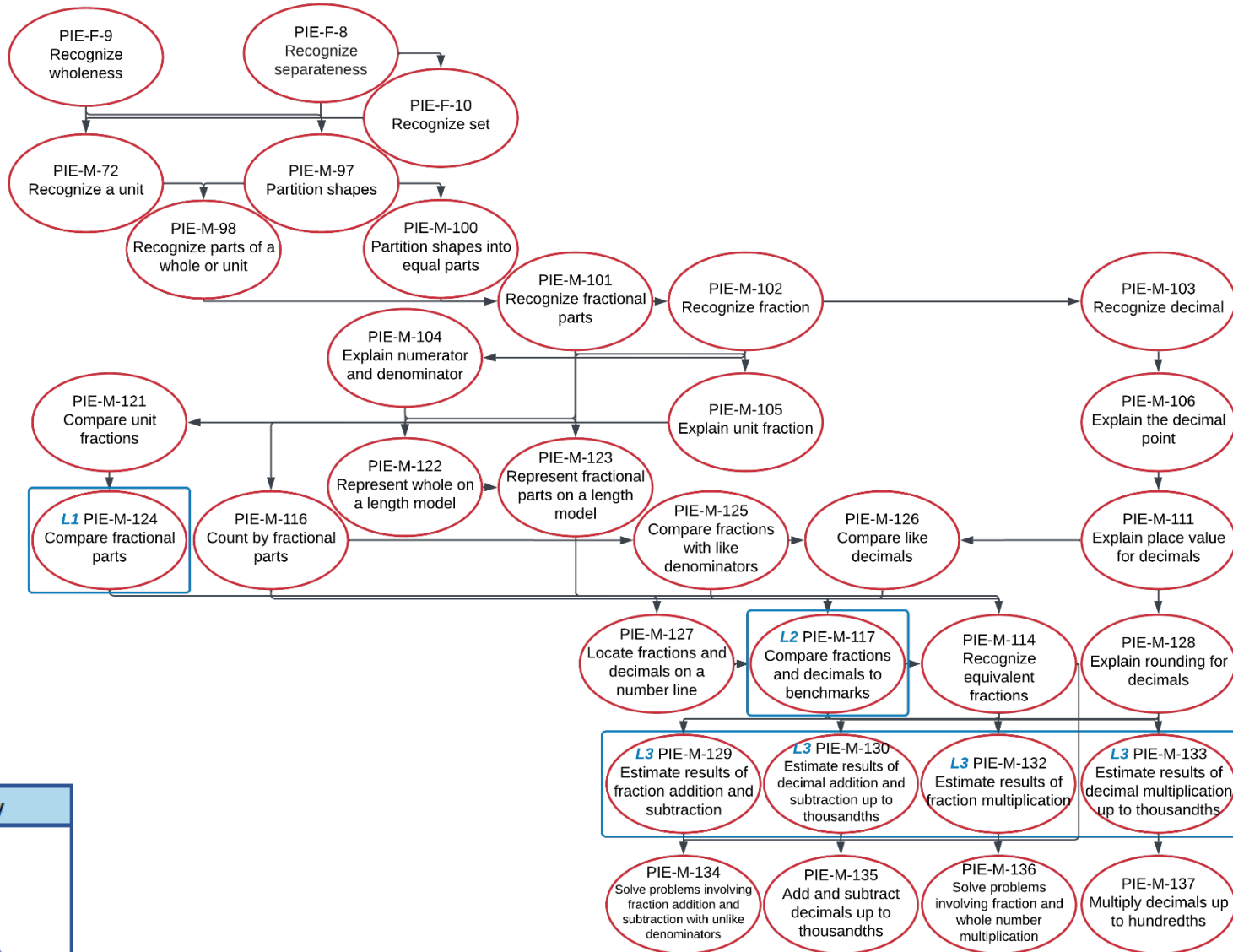**5.NF.B.** Perform operations and solve problems with fractions and decimals.
  5. Justify the reasonableness of a product when multiplying with fractions.
    b) Explain why multiplying a given number by a fraction greater than 1 results in a product larger than the given number.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Explain unit fraction (1/n), and count by fractional parts.

↓

Level 2
Express fraction as a product of a whole number and a unit fraction.

↓

Level 3
Explain why multiplying a given number by a fraction greater than one results in a product larger than the given number.

PIE.5.NF.B.5b Learning Pathway Map View

PIE-F-9
Recognize
wholeness

PIE-F-8
Recognize
separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a
whole or unit

PIE-M-100
Partition shapes into
equal parts

PIE-M-101
Recognize fractional
parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator
and denominator

PIE-M-121
Compare unit
fractions

L1 PIE-M-105
Explain unit fraction

L1 PIE-M-116
Count by fractional
parts

PIE-M-108
Represent whole on
an area model

PIE-M-110
Represent fractional
parts on an area
model

PIE-M-125
Compare fractions
with like
denominators

PIE-M-124
Compare fractional
parts

PIE-M-114
Recognize equivalent
fractions

PIE-M-143
Convert between
improper fractions
and mixed
numbers

PIE-M-144
Decompose fractions
into sums of unit
fractions

PIE-M-145
Represent equivalent
fractions with area
models

PIE-M-146
Represent fraction
multiplication with
area models

L2 PIE-M-147
Explain fraction as
product of whole number
and unit fraction

PIE-M-148
Explain fraction
multiplication

PIE-M-136
Solve problems
involving fraction and
whole number
multiplication

L3 PIE-M-141
Explain size of fraction
product when a factor
is greater than 1

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate
tested nodes

PIE.5.NF.B.5c

Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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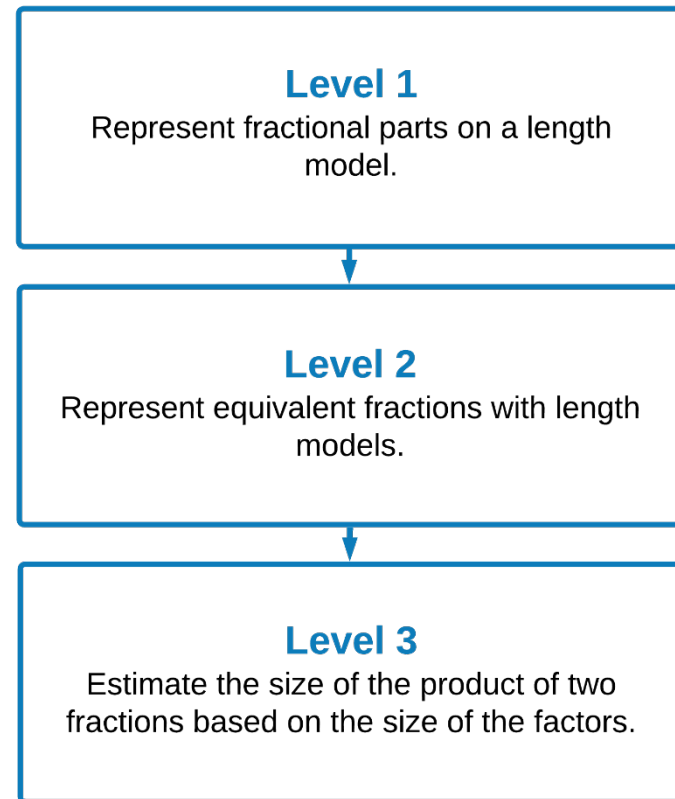
Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.
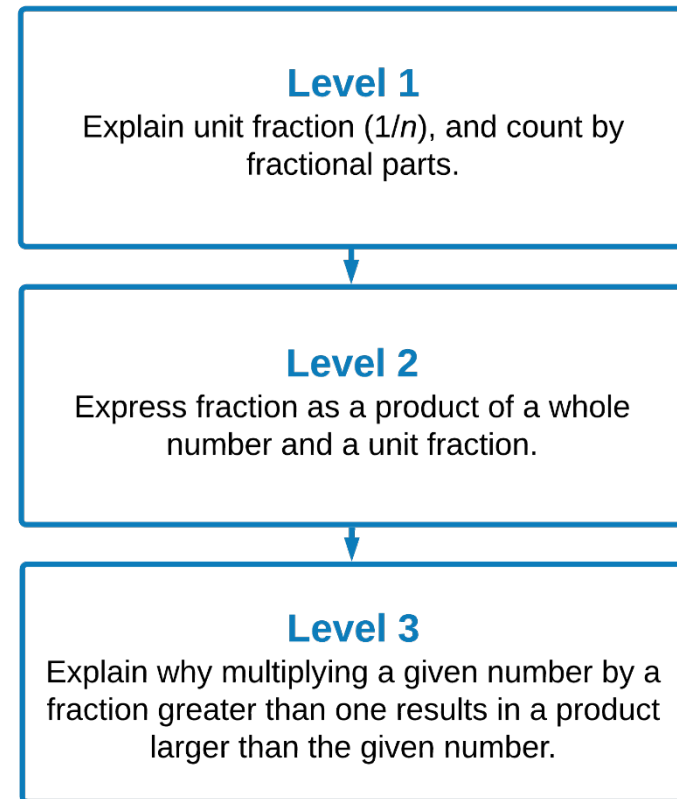
  5. Justify the reasonableness of a product when multiplying with fractions.

    c) Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Express fraction as division.

↓

Level 2
Express division as fraction multiplication.

↓

Level 3
Explain why multiplying a given number by a fraction less than one results in a product smaller than the given number.

PIE.5.NF.B.5c Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator and denominator

PIE-M-121
Compare unit fractions

PIE-M-105
Explain unit fraction

PIE-M-108
Represent whole on an area model

PIE-M-110
Represent fractional parts on an area model

PIE-M-125
Compare fractions with like denominators

PIE-M-116
Count by fractional parts

PIE-M-124
Compare fractional parts

PIE-M-114
Recognize equivalent fractions

PIE-M-143
Convert between improper fractions and mixed numbers

L1 PIE-M-150
Explain fraction as division

PIE-M-144
Decompose fractions into sums of unit fractions

PIE-M-145
Represent equivalent fractions with area models

PIE-M-146
Represent fraction multiplication with area models

L2 PIE-M-149
Explain division as fraction multiplication

PIE-M-147
Explain fraction as product of whole number and unit fraction

PIE-M-148
Explain fraction multiplication

PIE-M-136
Solve problems involving fraction and whole number multiplication

L3 PIE-M-142
Explain size of fraction product when a factor is less than 1

Map Key

L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.B.5d
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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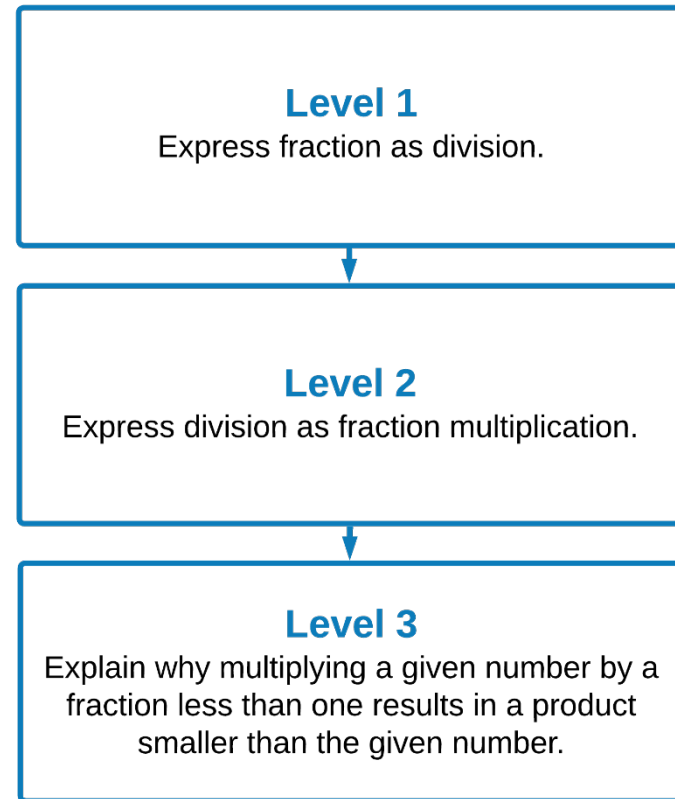
Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.

  5. Justify the reasonableness of a product when multiplying with fractions.

    d) Explain why multiplying the numerator and denominator by the same number is equivalent to multiplying the fraction by 1.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Represent equivalent fractions by multiplying or dividing the fraction by a fraction form of one.

↓

Level 2
Express fraction in equivalent forms.

↓

Level 3
Explain why multiplying the numerator and denominator by the same number is equivalent to multiplying the fraction by one.

PIE.5.NF.B.5d Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator and denominator

PIE-M-121
Compare unit fractions

PIE-M-105
Explain unit fraction

PIE-M-108
Represent whole on an area model

PIE-M-110
Represent fractional parts on an area model

PIE-M-125
Compare fractions with like denominators

PIE-M-116
Count by fractional parts

PIE-M-124
Compare fractional parts

PIE-M-114
Recognize equivalent fractions

PIE-M-144
Decompose fractions into sums of unit fractions

PIE-M-147
Explain fraction as product of whole number and unit fraction

PIE-M-148
Explain fraction multiplication

PIE-M-145
Represent equivalent fractions with area models

L1 PIE-M-151
Represent equivalent fractions by multiplying or dividing

L2 PIE-M-152
Explain simplest terms

L3 PIE-M-153
Relate the principle of fraction equivalence to multiplying by 1

PIE-M-136
Solve problems involving fraction and whole number multiplication

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.B.6
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.

Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.
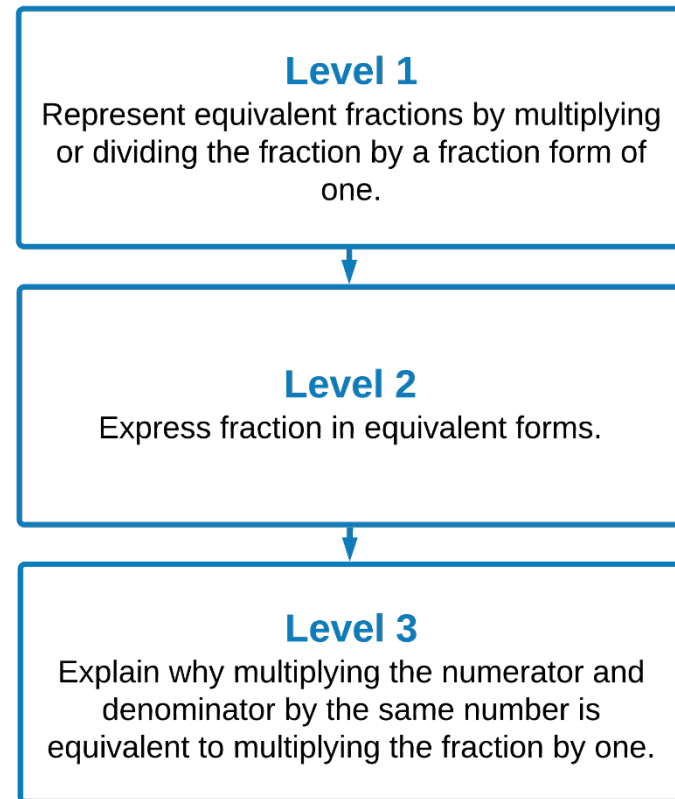
  6. Solve problems involving addition and subtraction of fractions and mixed numbers with unlike denominators, and justify the solution.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Add and subtract fractional parts.

↓

Level 2
Solve problems involving fraction addition and subtraction with like denominators.

↓

Level 3
Solve problems involving addition and subtraction of fractions, including mixed numbers, with unlike denominators, and justify the solution.

PIE.5.NF.B.6 Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator and denominator

PIE-M-105
Explain unit fraction

PIE-M-122
Represent whole on a length model

PIE-M-123
Represent fractional parts on a length model

PIE-M-125
Compare fractions with like denominators

PIE-M-116
Count by fractional parts

PIE-M-127
Locate fractions and decimals on a number line

PIE-M-117
Compare fractions and decimals to benchmarks

PIE-M-114
Recognize equivalent fractions

PIE-M-144
Decompose fractions into sums of unit fractions

L1 PIE-M-154
Add and subtract fractional parts

PIE-M-129
Estimate results of fraction addition and subtraction

PIE-M-143
Convert between improper fractions and mixed numbers

PIE-M-155
Explain fraction addition and subtraction

PIE-M-156
Explain common denominator

L2 PIE-M-157
Solve problems involving fraction addition and subtraction with like denominators

L3 PIE-M-134
Solve problems involving fraction addition and subtraction with unlike denominators

L3 PIE-M-158
Evaluate solutions of problems with fraction addition and subtraction with unlike denominators

PIE-M-159
Solve multi-step problems with variables and rational numbers

PIE-M-160
Evaluate solutions of multi-step problems with variables and rational numbers

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.NF.B.7a

Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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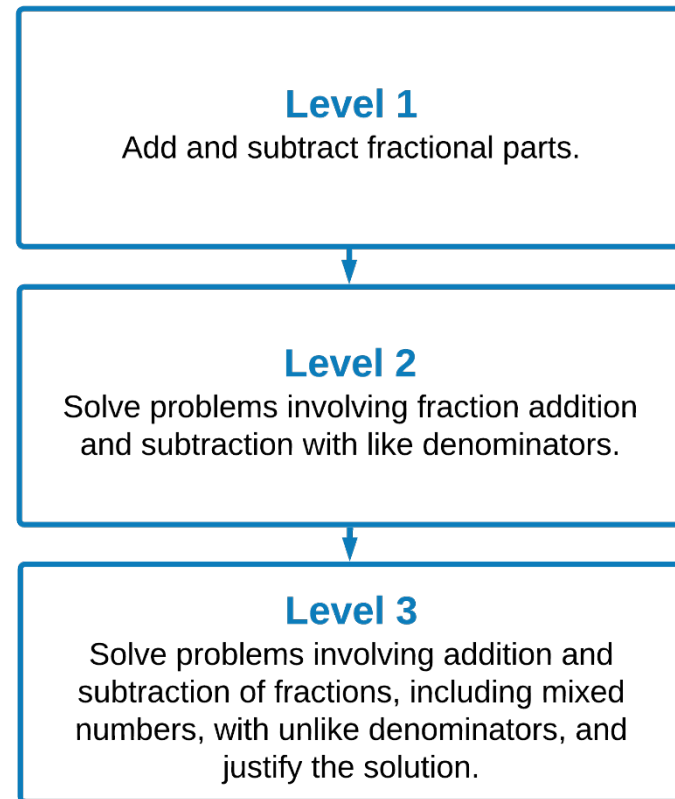
Learning Target

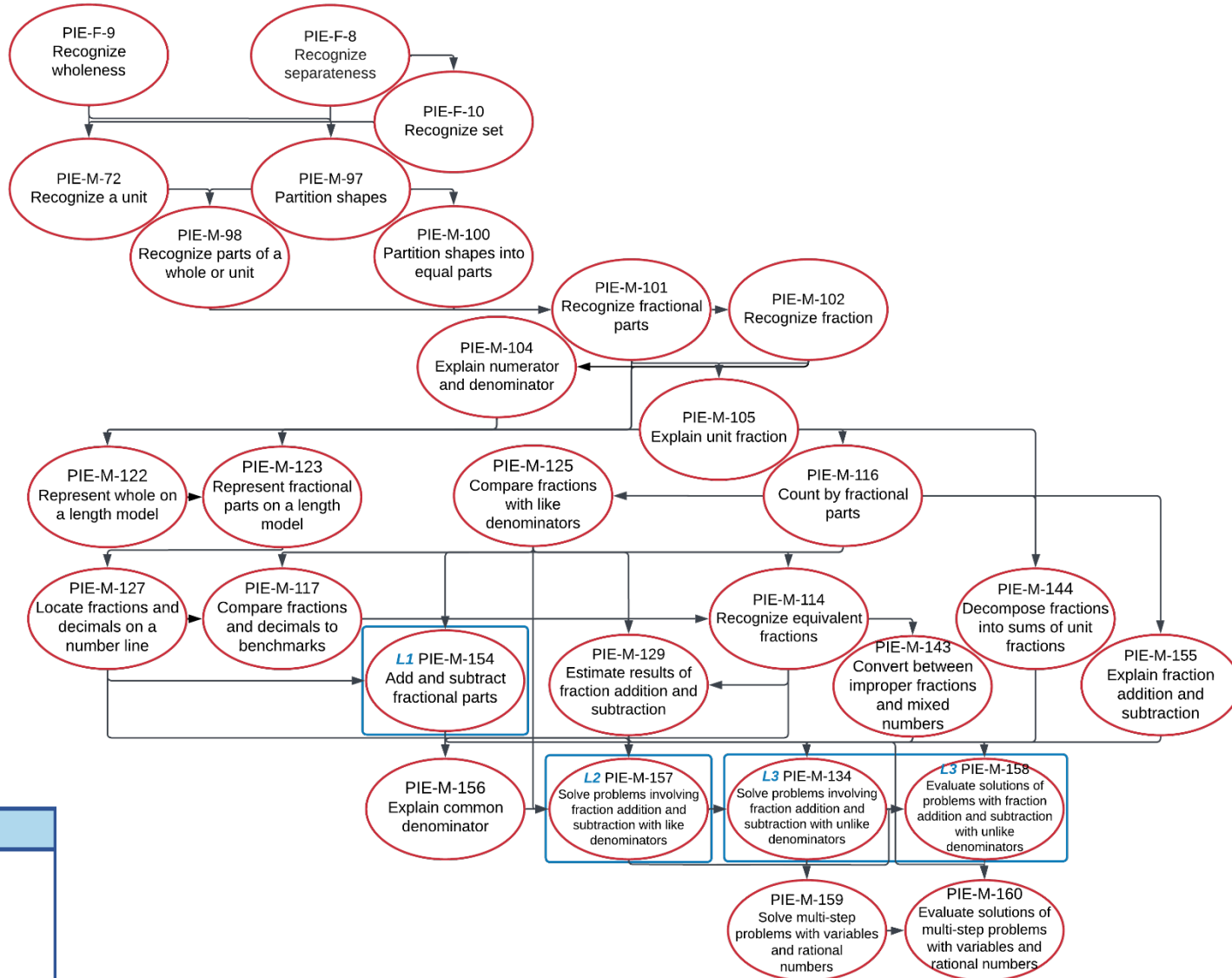**5.NF.B.** Perform operations and solve problems with fractions and decimals.

   7. Extend the concept of multiplication to multiply a fraction or whole number by a fraction.

   a) Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Represent fractional parts on an area model.

↓

Level 2
Determine the area of a rectangle with fractional side lengths.

↓

Level 3
Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths.

PIE.5.NF.B.7a Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator and denominator

PIE-M-121
Compare unit fractions

PIE-M-105
Explain unit fraction

L1 PIE-M-110
Represent fractional parts on an area model

PIE-M-108
Represent whole on an area model

PIE-M-125
Compare fractions with like denominators

PIE-M-116
Count by fractional parts

PIE-M-124
Compare fractional parts

PIE-M-114
Recognize equivalent fractions

PIE-M-143
Convert between improper fractions and mixed numbers

PIE-M-146
Represent fraction multiplication with area models

PIE-M-144
Decompose fractions into sums of unit fractions

PIE-M-145
Represent equivalent fractions with area models

L2 PIE-M-161
Determine area of a rectangle with fractional side lengths

PIE-M-147
Explain fraction as product of whole number and unit fraction

PIE-M-148
Explain fraction mutiplication

L3 PIE-M-162
Relate fraction multiplication to area of rectangles with fractional side lengths

PIE-M-136
Solve problems involving fraction and whole number multiplication

PIE-M-163
Solve problems involving multiplication of fractions less than 1

Map Key

L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.B.7b
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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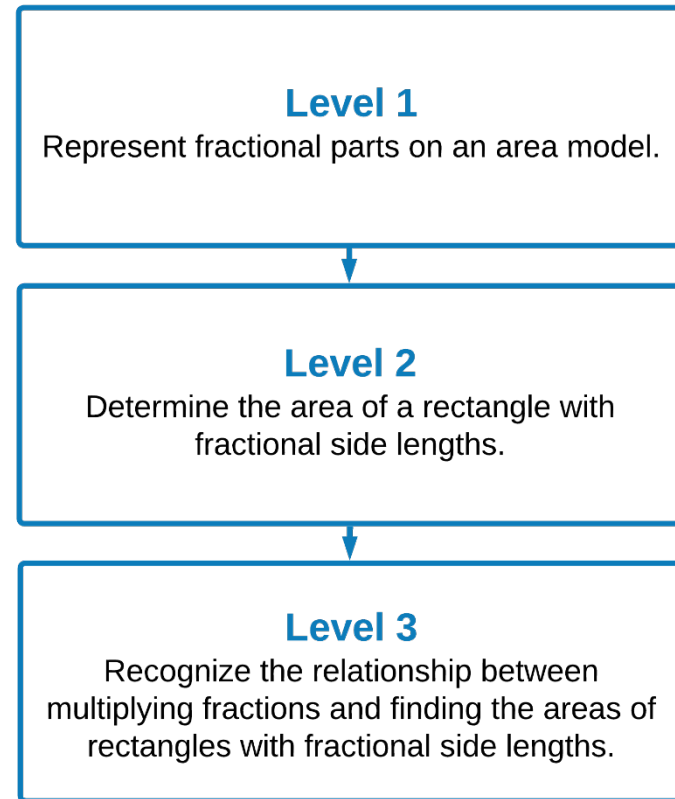
Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.

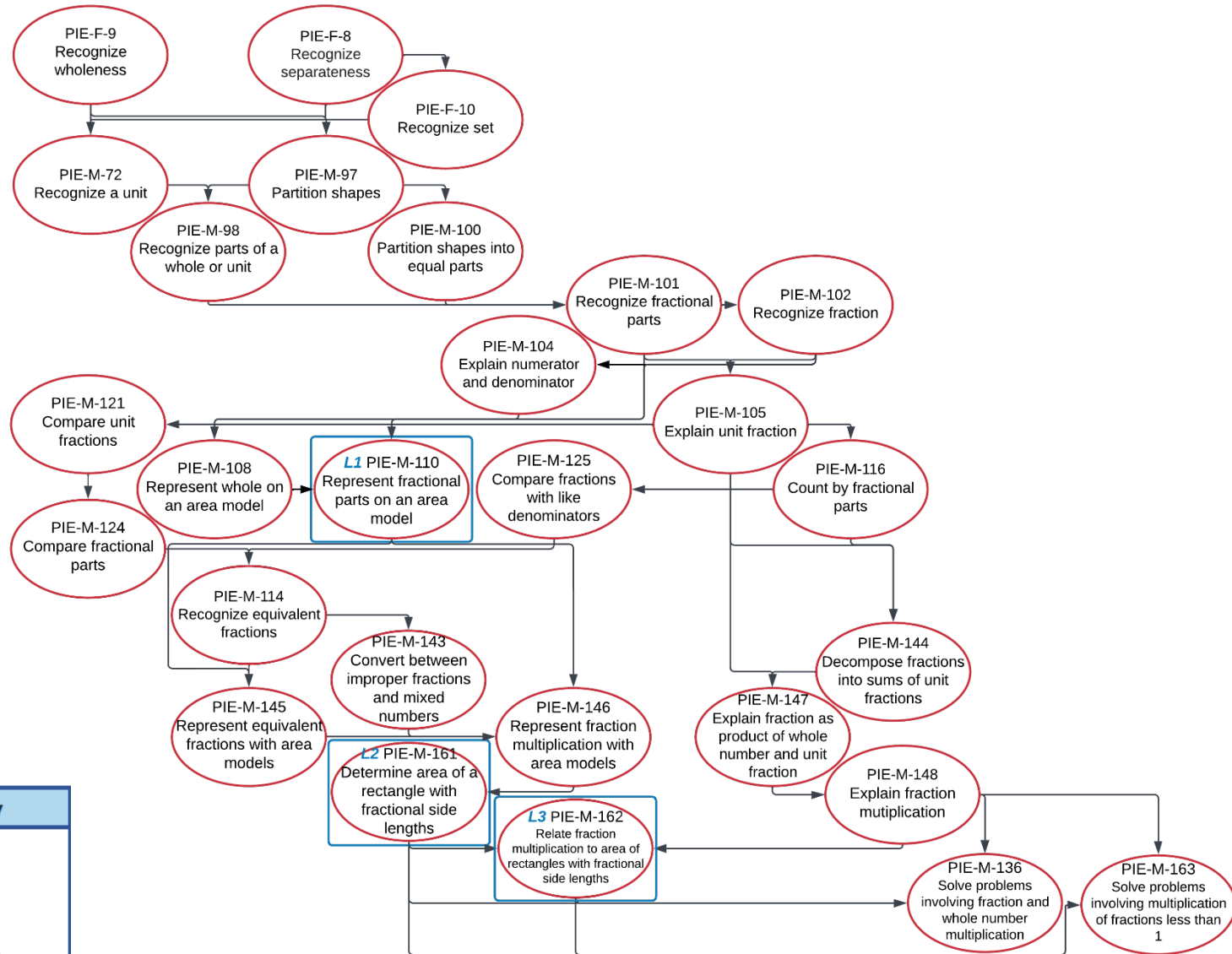   **7.** Extend the concept of multiplication to multiply a fraction or whole number by a fraction.

   b) Calculate and interpret the product of a fraction by a whole number and a whole number by a fraction.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Represent fractional parts on an area model.

Level 2
Represent whole number and fraction multiplication using area models.

Level 3
Calculate and interpret the product of a fraction by a whole number and a whole number by a fraction.

PIE.5.NF.B.7b Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator and denominator

PIE-M-121
Compare unit fractions

PIE-M-105
Explain unit fraction

PIE-M-108
Represent whole on an area model

L1 PIE-M-110
Represent fractional parts on an area model

PIE-M-125
Compare fractions with like denominators

PIE-M-116
Count by fractional parts

PIE-M-124
Compare fractional parts

PIE-M-114
Recognize equivalent fractions

PIE-M-143
Convert between improper fractions and mixed numbers

PIE-M-150
Explain fraction as division

PIE-M-144
Decompose fractions into sums of unit fractions

PIE-M-132
Estimate results of fraction multiplication

PIE-M-145
Represent equivalent fractions with area models

L2 PIE-M-146
Represent fraction multiplication with area models

PIE-M-149
Explain division as fraction multiplication

PIE-M-147
Explain fraction as product of whole number and unit fraction

PIE-M-148
Explain fraction multiplication

L3 PIE-M-164
Interpret the product of whole number and fraction

PIE-M-141
Explain size of fraction product when a factor is greater than 1

PIE-M-142
Explain size of fraction product when a factor is less than 1

L3 PIE-M-136
Solve problems involving fraction and whole number multiplication

PIE-M-159
Solve multi-step problems with variables and rational numbers

PIE-M-160
Evaluate solutions of multi-step problems with variables and rational numbers

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.B.7c
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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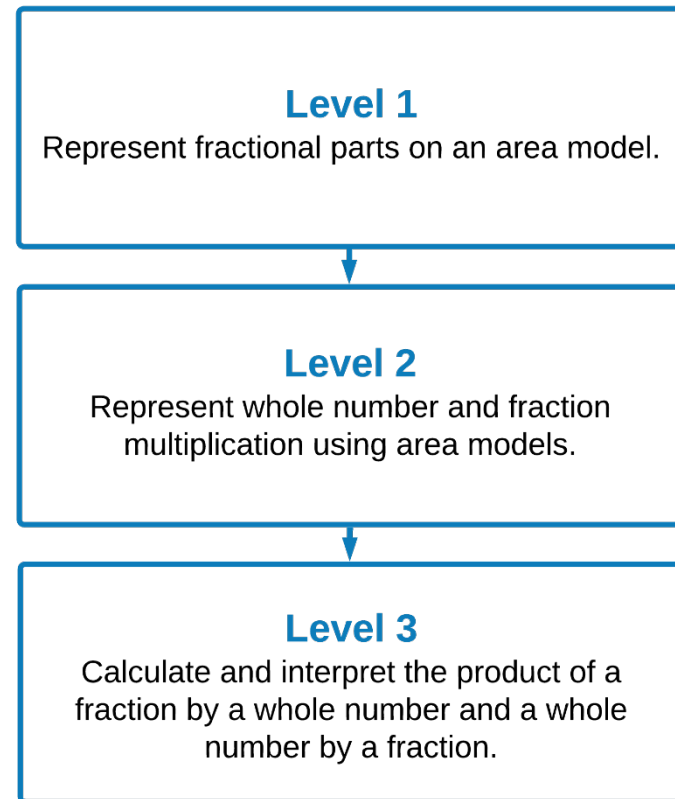
Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.

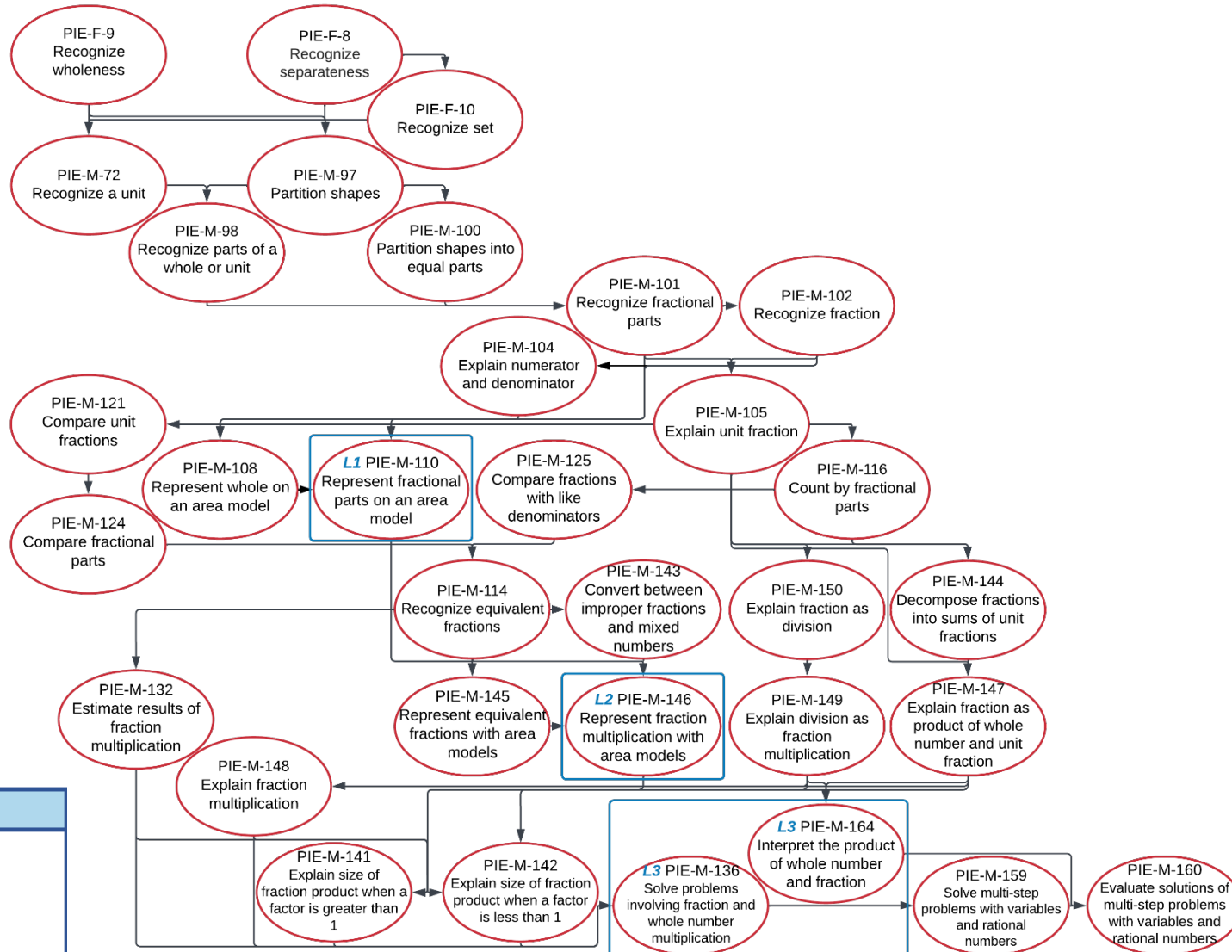   **7.** Extend the concept of multiplication to multiply a fraction or whole number by a fraction.

      c) Calculate and interpret the product of two fractions less than one.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Represent fractional parts on an area model.

↓

Level 2
Represent whole number and fraction multiplication using area models.

↓

Level 3
Calculate and interpret the product of two fractions less than one.

PIE.5.NF.B.7c Learning Pathway Map View

PIE-F-9 Recognize wholeness

PIE-F-8 Recognize separateness

PIE-F-10 Recognize set

PIE-M-72 Recognize a unit

PIE-M-97 Partition shapes

PIE-M-98 Recognize parts of a whole or unit

PIE-M-100 Partition shapes into equal parts

PIE-M-101 Recognize fractional parts

PIE-M-102 Recognize fraction

PIE-M-104 Explain numerator and denominator

PIE-M-121 Compare unit fractions

PIE-M-105 Explain unit fraction

PIE-M-108 Represent whole on an area model

L1 PIE-M-110 Represent fractional parts on an area model

PIE-M-125 Compare fractions with like denominators

PIE-M-116 Count by fractional parts

PIE-M-124 Compare fractional parts

PIE-M-114 Recognize equivalent fractions

PIE-M-143 Convert between improper fractions and mixed numbers

PIE-M-150 Explain fraction as division

PIE-M-144 Decompose fractions into sums of unit fractions

PIE-M-132 Estimate results of fraction multiplication

PIE-M-145 Represent equivalent fractions with area models

L2 PIE-M-146 Represent fraction multiplication with area models

PIE-M-149 Explain division as fraction multiplication

PIE-M-147 Explain fraction as product of whole number and unit fraction

PIE-M-148 Explain fraction multiplication

L3 PIE-M-165 Interpret the product of fractions less than 1

PIE-M-141 Explain size of fraction product when a factor is greater than 1

PIE-M-142 Explain size of fraction product when a factor is less than 1

L3 PIE-M-163 Solve problems involving multiplication of fractions less than 1

PIE-M-159 Solve multi-step problems with variables and rational numbers

PIE-M-160 Evaluate solutions of multi-step problems with variables and rational numbers

Map Key

L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.B.8a
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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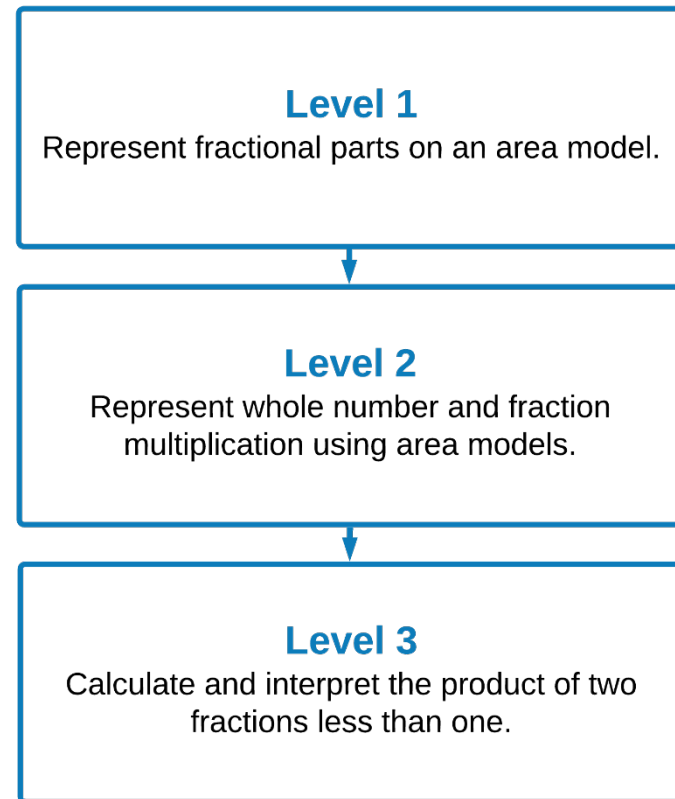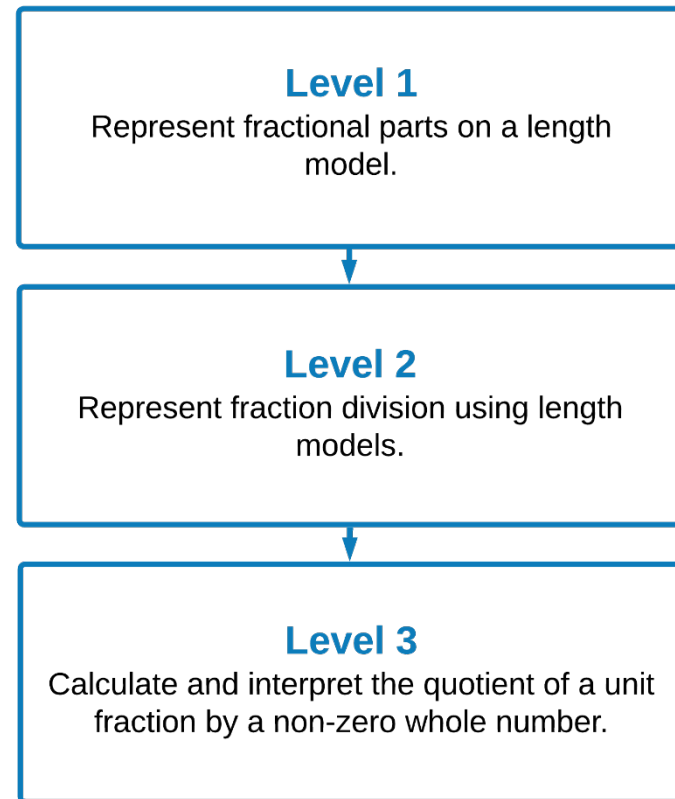
Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.

   8. Extend the concept of division to divide unit fractions and whole numbers by using visual fraction models and equations.

      a) Calculate and interpret the quotient of a unit fraction by a non-zero whole number.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Represent fractional parts on a length model.

↓

Level 2
Represent fraction division using length models.

↓

Level 3
Calculate and interpret the quotient of a unit fraction by a non-zero whole number.

PIE.5.NF.B.8a Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator and denominator

PIE-M-121
Compare unit fractions

PIE-M-105
Explain unit fraction

PIE-M-122
Represent whole on a length model

L1 PIE-M-123
Represent fractional parts on a length model

PIE-M-125
Compare fractions with like denominators

PIE-M-116
Count by fractional parts

PIE-M-124
Compare fractional parts

PIE-M-114
Recognize equivalent fractions

PIE-M-150
Explain fraction as division

PIE-M-144
Decompose fractions into sums of unit fractions

PIE-M-138
Represent equivalent fractions with length models

L2 PIE-M-173
Represent fraction division with length models

PIE-M-149
Explain division as fraction multiplication

PIE-M-147
Explain fraction as product of whole number and unit fraction

PIE-M-166
Estimate results of fraction division

PIE-M-148
Explain fraction multiplication

PIE-M-167
Explain division of unit fractions by whole numbers

L3 PIE-M-169
Interpret the quotient of unit fraction by whole number

L3 PIE-M-168
Solve problems involving division of unit fractions by whole numbers

PIE-M-159
Solve multi-step problems with variables and rational numbers

PIE-M-160
Evaluate solutions of multi-step problems with variables and rational numbers

Map Key

L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.NF.B.8b
Mathematics
Number Sense and Operations in Fractions (NF)
Grade 5

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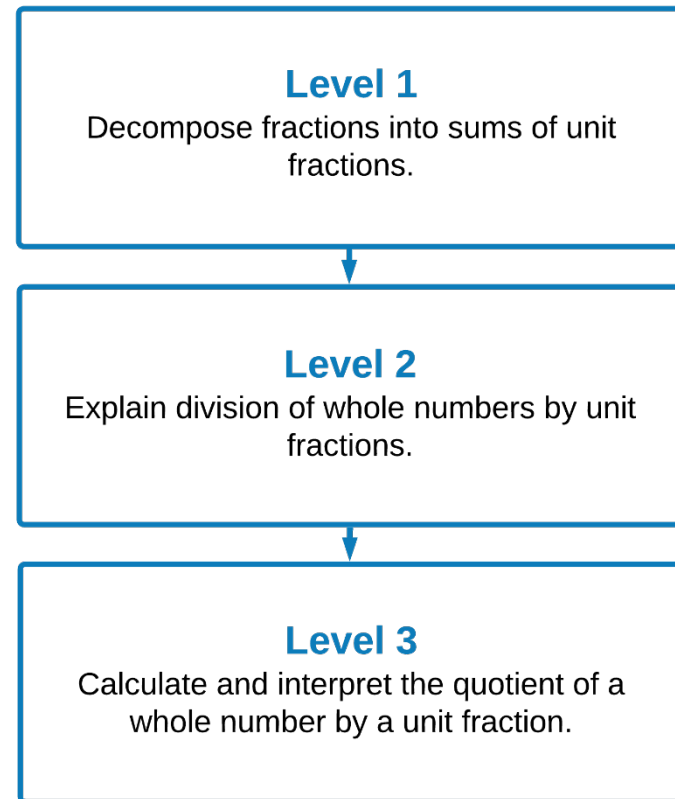
Learning Target

5.NF.B. Perform operations and solve problems with fractions and decimals.

  8. Extend the concept of division to divide unit fractions and whole numbers by using visual fraction models and equations.

    b) Calculate and interpret the quotient of a whole number by a unit fraction.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Decompose fractions into sums of unit fractions.

↓

Level 2
Explain division of whole numbers by unit fractions.

↓

Level 3
Calculate and interpret the quotient of a whole number by a unit fraction.

PIE.5.NF.B.8b Learning Pathway Map View

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-10
Recognize set

PIE-M-72
Recognize a unit

PIE-M-97
Partition shapes

PIE-M-98
Recognize parts of a whole or unit

PIE-M-100
Partition shapes into equal parts

PIE-M-101
Recognize fractional parts

PIE-M-102
Recognize fraction

PIE-M-104
Explain numerator and denominator

PIE-M-121
Compare unit fractions

PIE-M-105
Explain unit fraction

PIE-M-122
Represent whole on a length model

PIE-M-123
Represent fractional parts on a length model

PIE-M-125
Compare fractions with like denominators

PIE-M-116
Count by fractional parts

PIE-M-124
Compare fractional parts

PIE-M-114
Recognize equivalent fractions

PIE-M-150
Explain fraction as division

L1 PIE-M-144
Decompose fractions into sums of unit fractions

PIE-M-138
Represent equivalent fractions with length models

PIE-M-173
Represent fraction division with length models

PIE-M-149
Explain division as fraction multiplication

L2 PIE-M-170
Explain division of whole numbers by unit fractions

PIE-M-166
Estimate results of fraction division

L3 PIE-M-171
Solve problems involving division of whole numbers by unit fractions

L3 PIE-M-172
Interpret the quotient of whole number by unit fraction

PIE-M-159
Solve multi-step problems with variables and rational numbers

PIE-M-160
Evaluate solutions of multi-step problems with variables and rational numbers

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.RA.A.1a
Mathematics
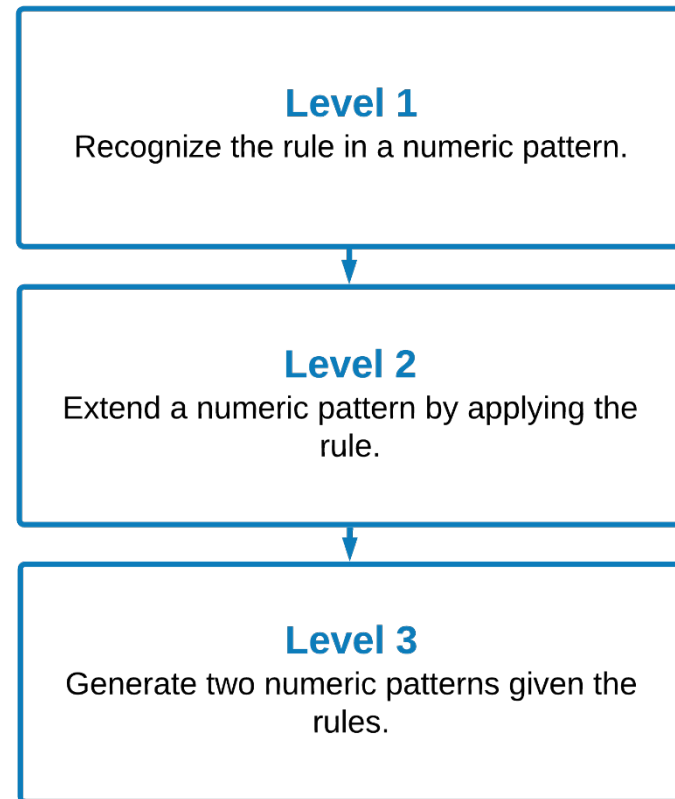## Relationships and Algebraic Thinking (RA)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
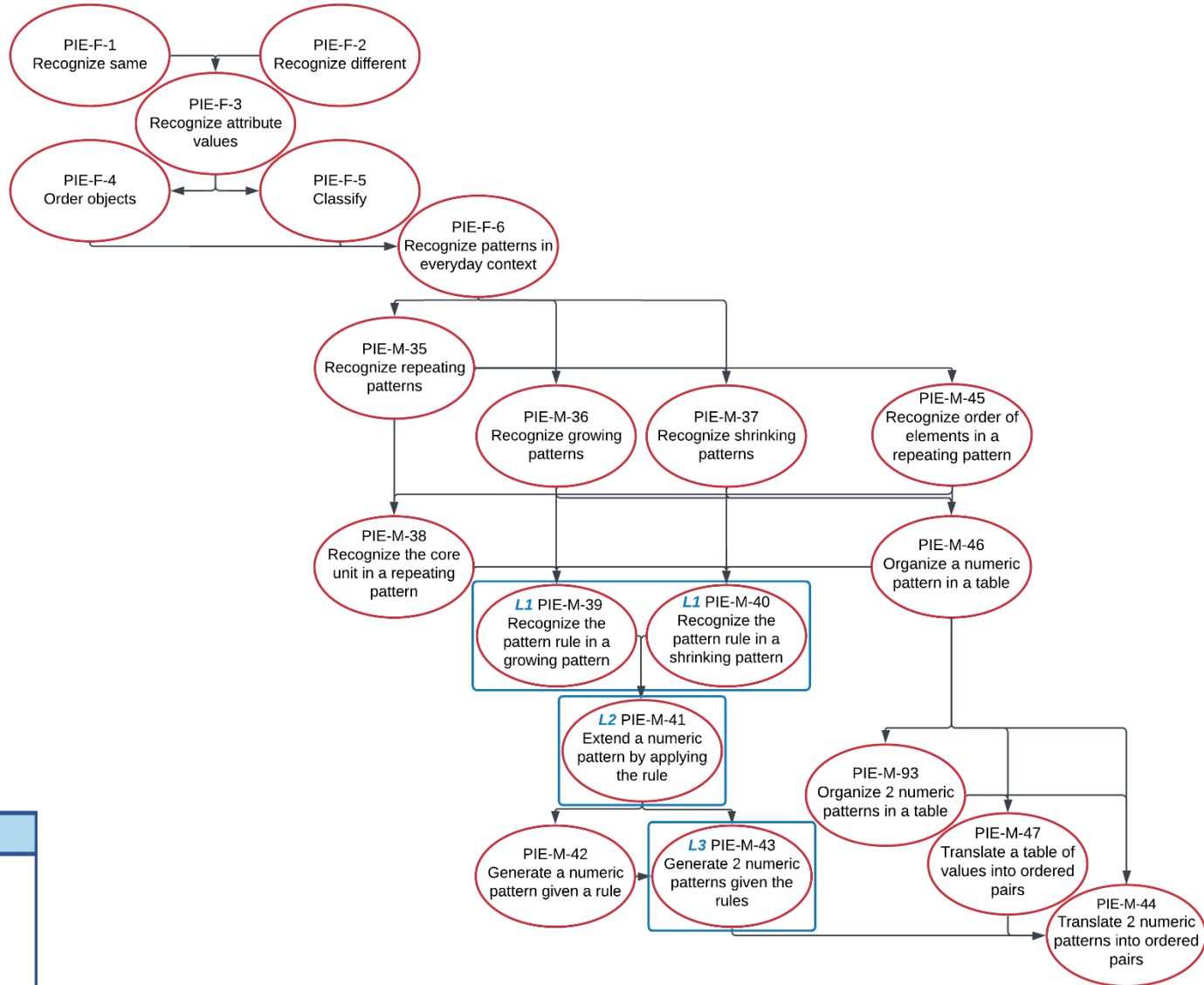
Learning Target

5.RA.A. Represent and analyze patterns and relationships.
  1. Investigate the relationship between two numeric patterns.
    a) Generate two numeric patterns given two rules.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Recognize the rule in a numeric pattern.

↓

Level 2
Extend a numeric pattern by applying the rule.

↓

Level 3
Generate two numeric patterns given the rules.

PIE.5.RA.A.1a Learning Pathway Map View

PIE-F-1
Recognize same

PIE-F-2
Recognize different

PIE-F-3
Recognize attribute values

PIE-F-4
Order objects

PIE-F-5
Classify

PIE-F-6
Recognize patterns in everyday context

PIE-M-35
Recognize repeating patterns

PIE-M-36
Recognize growing patterns

PIE-M-37
Recognize shrinking patterns

PIE-M-45
Recognize order of elements in a repeating pattern

PIE-M-38
Recognize the core unit in a repeating pattern

L1 PIE-M-39
Recognize the pattern rule in a growing pattern

L1 PIE-M-40
Recognize the pattern rule in a shrinking pattern

PIE-M-46
Organize a numeric pattern in a table

L2 PIE-M-41
Extend a numeric pattern by applying the rule

PIE-M-93
Organize 2 numeric patterns in a table

PIE-M-42
Generate a numeric pattern given a rule

L3 PIE-M-43
Generate 2 numeric patterns given the rules

PIE-M-47
Translate a table of values into ordered pairs

PIE-M-44
Translate 2 numeric patterns into ordered pairs

Map Key

L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.RA.A.1b
Mathematics
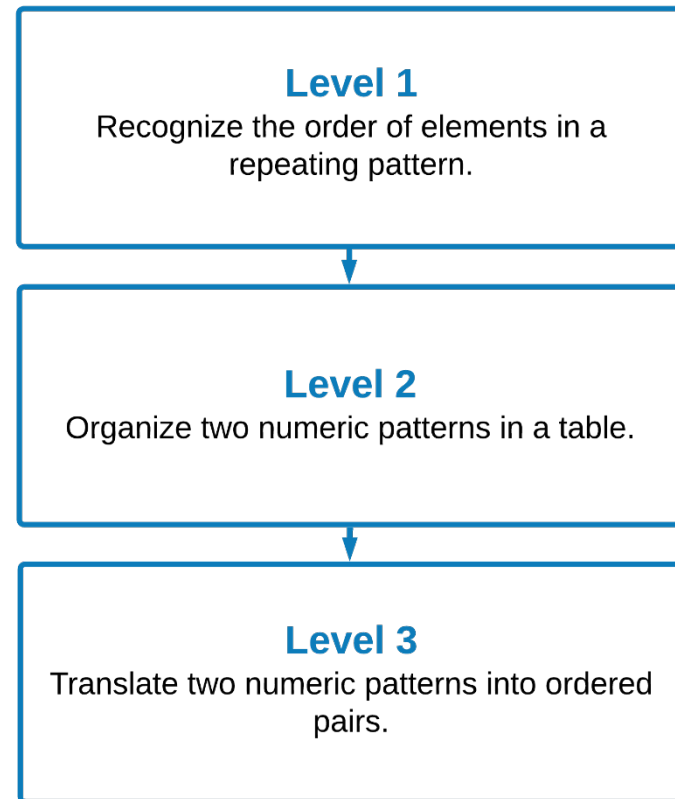## Relationships and Algebraic Thinking (RA)
Grade 5

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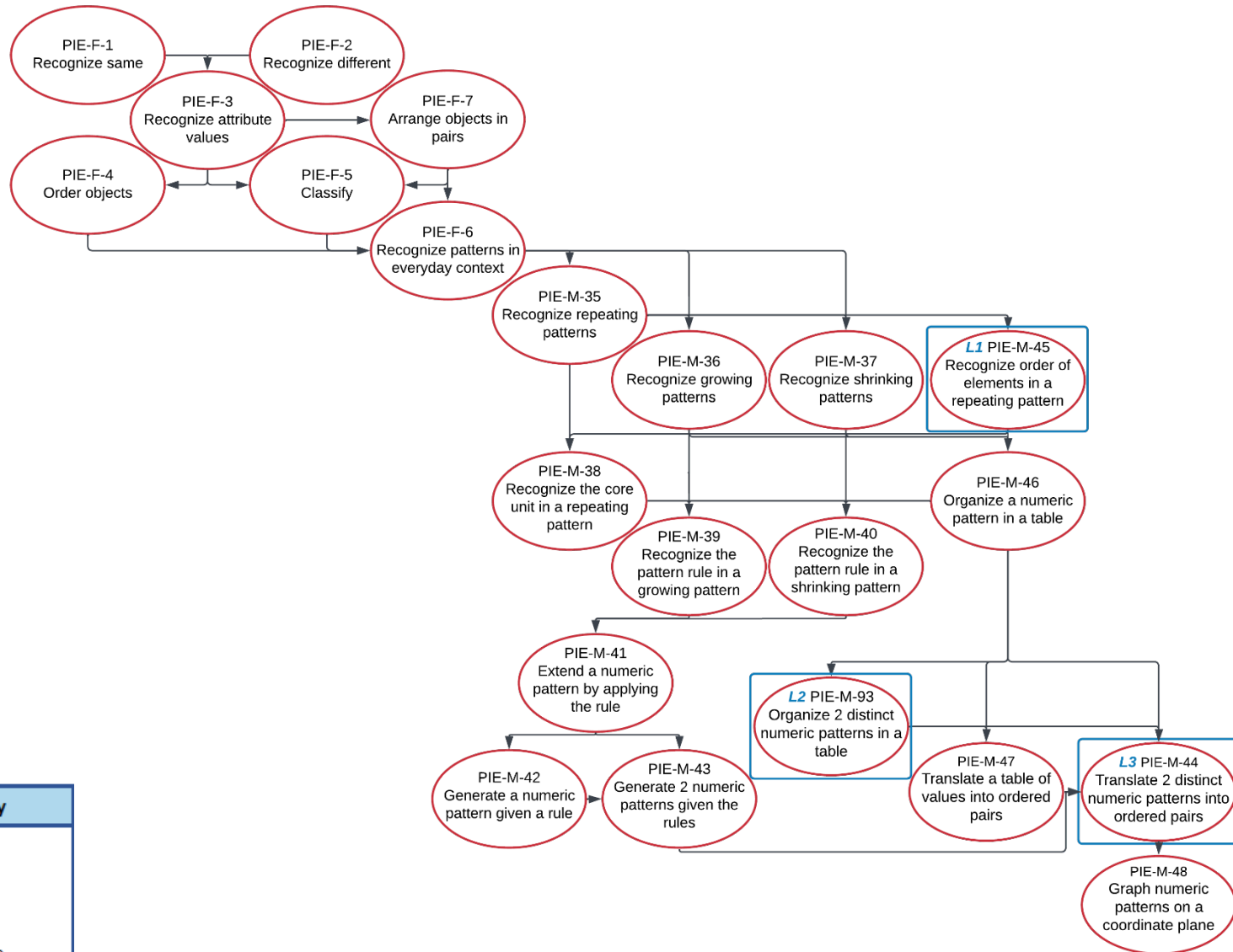
Learning Target

5.RA.A. Represent and analyze patterns and relationships.
  1. Investigate the relationship between two numeric patterns.
    b) Translate two numeric patterns into two sets of ordered pairs.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Recognize the order of elements in a repeating pattern.

↓

Level 2
Organize two numeric patterns in a table.

↓

Level 3
Translate two numeric patterns into ordered pairs.

PIE.5.RA.A.1b Learning Pathway Map View

```
PIE-F-1                    PIE-F-2
Recognize same             Recognize different

        PIE-F-3                PIE-F-7
        Recognize attribute    Arrange objects in
        values                 pairs

PIE-F-4                PIE-F-5
Order objects          Classify

                PIE-F-6
                Recognize patterns in
                everyday context

        PIE-M-35
        Recognize repeating
        patterns

                PIE-M-36           PIE-M-37           L1 PIE-M-45
                Recognize growing  Recognize shrinking Recognize order of
                patterns           patterns           elements in a
                                                      repeating pattern

PIE-M-38                                              PIE-M-46
Recognize the core                                   Organize a numeric
unit in a repeating                                  pattern in a table
pattern
        PIE-M-39           PIE-M-40
        Recognize the      Recognize the
        pattern rule in a  pattern rule in a
        growing pattern    shrinking pattern

PIE-M-41
Extend a numeric
pattern by applying   L2 PIE-M-93
the rule              Organize 2 distinct
                      numeric patterns in a
                      table

PIE-M-42          PIE-M-43           PIE-M-47           L3 PIE-M-44
Generate a numeric Generate 2 numeric Translate a table of Translate 2 distinct
pattern given a rule patterns given the values into ordered numeric patterns into
                   rules            pairs              ordered pairs

                                                       PIE-M-48
                                                       Graph numeric
                                                       patterns on a
                                                       coordinate plane
```

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.RA.A.1c
Mathematics
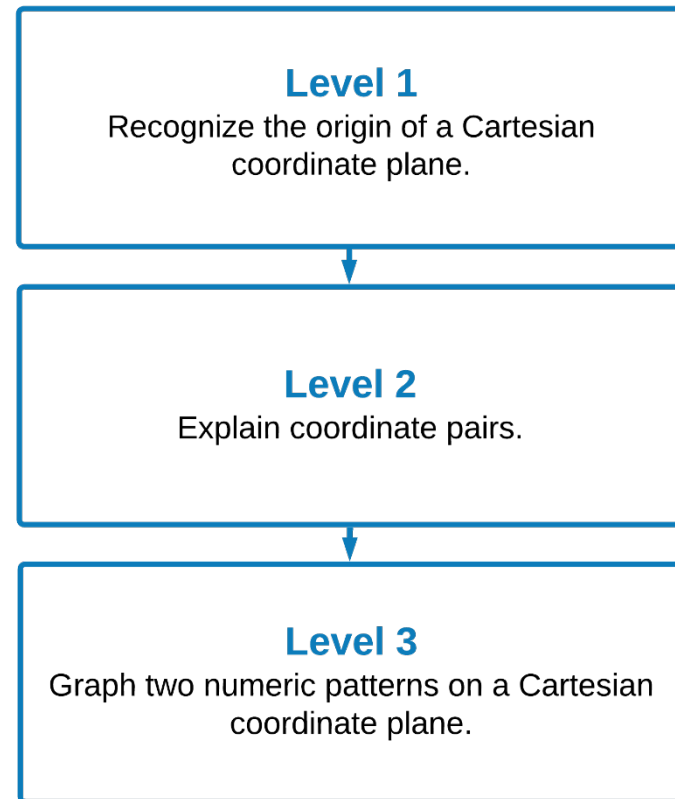## Relationships and Algebraic Thinking (RA)
Grade 5

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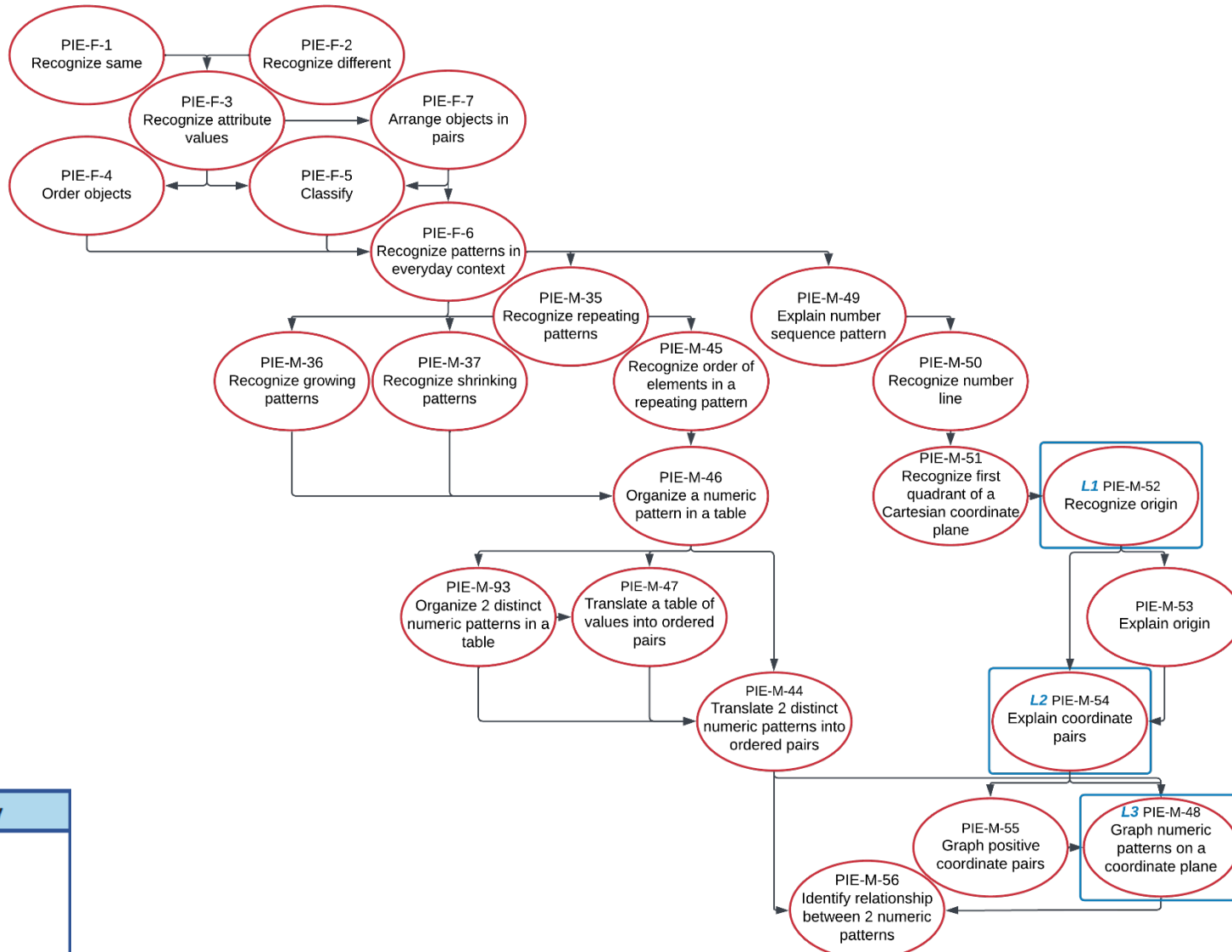
Learning Target

5.RA.A. Represent and analyze patterns and relationships.
  1. Investigate the relationship between two numeric patterns.
    c) Graph numeric patterns on the Cartesian coordinate plane.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Recognize the origin of a Cartesian coordinate plane.

↓

Level 2
Explain coordinate pairs.

↓

Level 3
Graph two numeric patterns on a Cartesian coordinate plane.

PIE.5.RA.A.1c Learning Pathway Map View

PIE-F-1
Recognize same

PIE-F-2
Recognize different

PIE-F-3
Recognize attribute values

PIE-F-7
Arrange objects in pairs

PIE-F-4
Order objects

PIE-F-5
Classify

PIE-F-6
Recognize patterns in everyday context

PIE-M-35
Recognize repeating patterns

PIE-M-49
Explain number sequence pattern

PIE-M-36
Recognize growing patterns

PIE-M-37
Recognize shrinking patterns

PIE-M-45
Recognize order of elements in a repeating pattern

PIE-M-50
Recognize number line

PIE-M-46
Organize a numeric pattern in a table

PIE-M-51
Recognize first quadrant of a Cartesian coordinate plane

L1 PIE-M-52
Recognize origin

PIE-M-93
Organize 2 distinct numeric patterns in a table

PIE-M-47
Translate a table of values into ordered pairs

PIE-M-53
Explain origin

PIE-M-44
Translate 2 distinct numeric patterns into ordered pairs

L2 PIE-M-54
Explain coordinate pairs

PIE-M-55
Graph positive coordinate pairs

L3 PIE-M-48
Graph numeric patterns on a coordinate plane

PIE-M-56
Identify relationship between 2 numeric patterns

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.RA.A.1d
Mathematics
Relationships and Algebraic Thinking (RA)
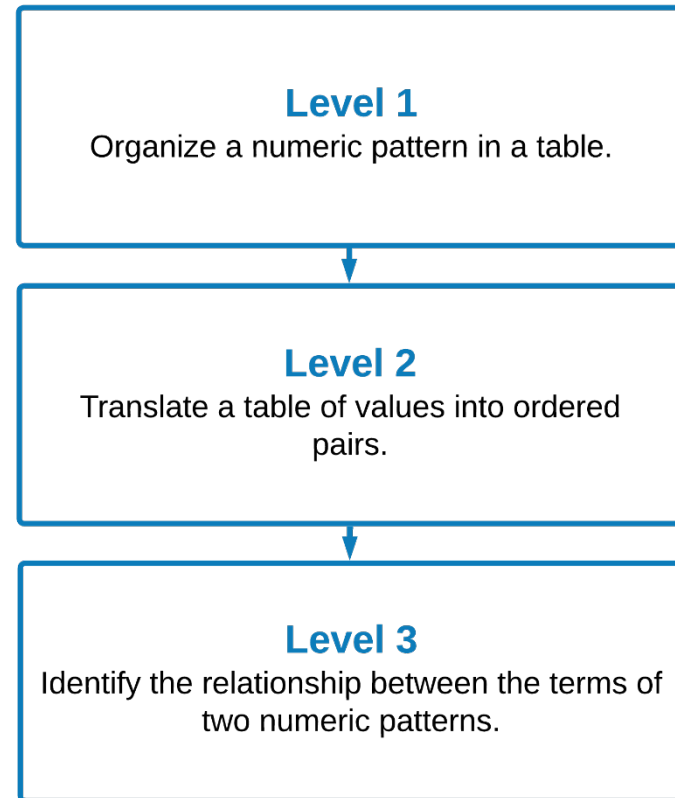## Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
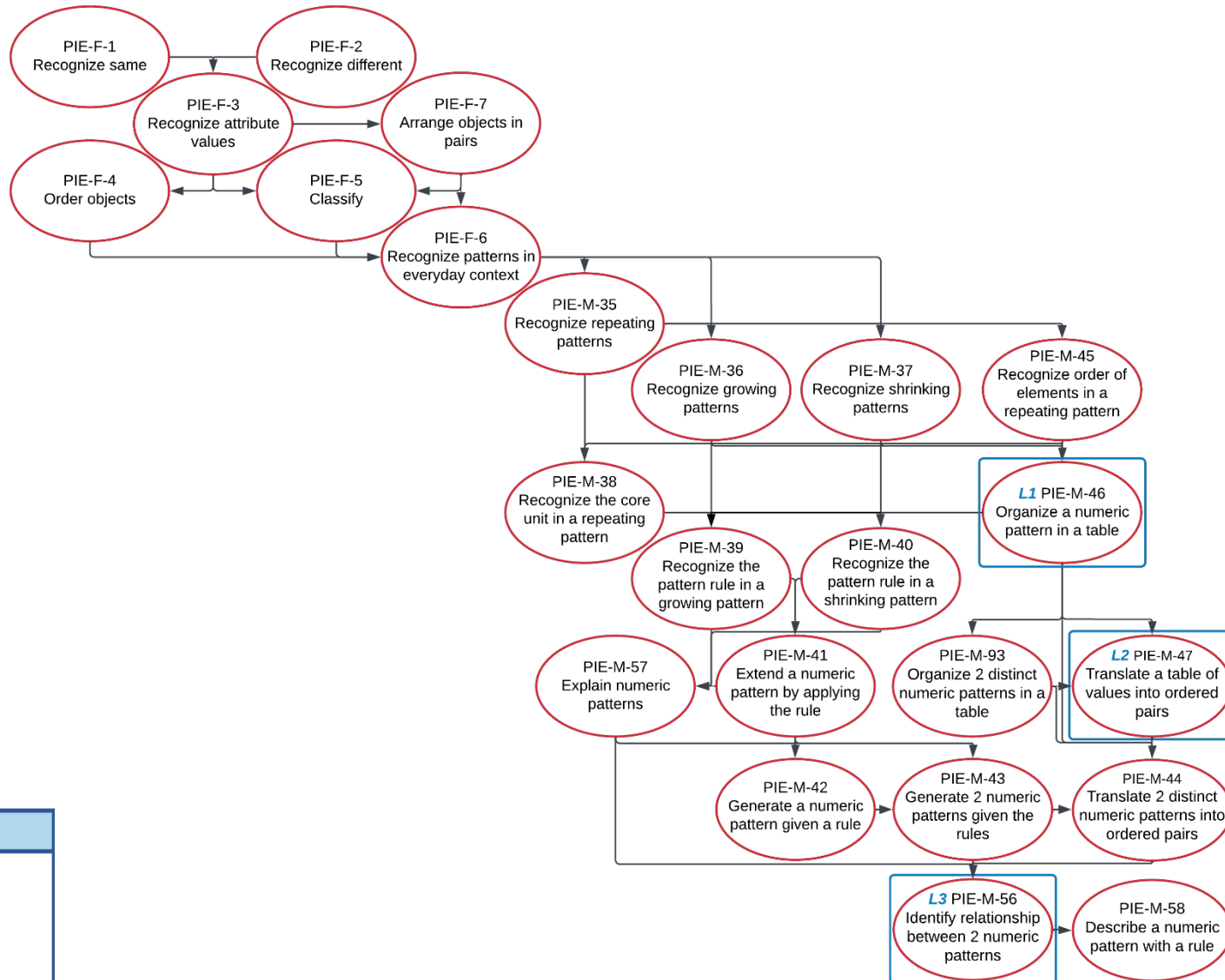
Learning Target

5.RA.A. Represent and analyze patterns and relationships.
  1. Investigate the relationship between two numeric patterns.
    d) Identify the relationship between two numeric patterns.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Organize a numeric pattern in a table.

↓

Level 2
Translate a table of values into ordered pairs.

↓

Level 3
Identify the relationship between the terms of two numeric patterns.

PIE.5.RA.A.1d Learning Pathway Map View

PIE-F-1
Recognize same

PIE-F-2
Recognize different

PIE-F-3
Recognize attribute values

PIE-F-7
Arrange objects in pairs

PIE-F-4
Order objects

PIE-F-5
Classify

PIE-F-6
Recognize patterns in everyday context

PIE-M-35
Recognize repeating patterns

PIE-M-36
Recognize growing patterns

PIE-M-37
Recognize shrinking patterns

PIE-M-45
Recognize order of elements in a repeating pattern

PIE-M-38
Recognize the core unit in a repeating pattern

L1 PIE-M-46
Organize a numeric pattern in a table

PIE-M-39
Recognize the pattern rule in a growing pattern

PIE-M-40
Recognize the pattern rule in a shrinking pattern

PIE-M-57
Explain numeric patterns

PIE-M-41
Extend a numeric pattern by applying the rule

PIE-M-93
Organize 2 distinct numeric patterns in a table

L2 PIE-M-47
Translate a table of values into ordered pairs

PIE-M-42
Generate a numeric pattern given a rule

PIE-M-43
Generate 2 numeric patterns given the rules

PIE-M-44
Translate 2 distinct numeric patterns into ordered pairs

L3 PIE-M-56
Identify relationship between 2 numeric patterns

PIE-M-58
Describe a numeric pattern with a rule

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.RA.A.2
Mathematics
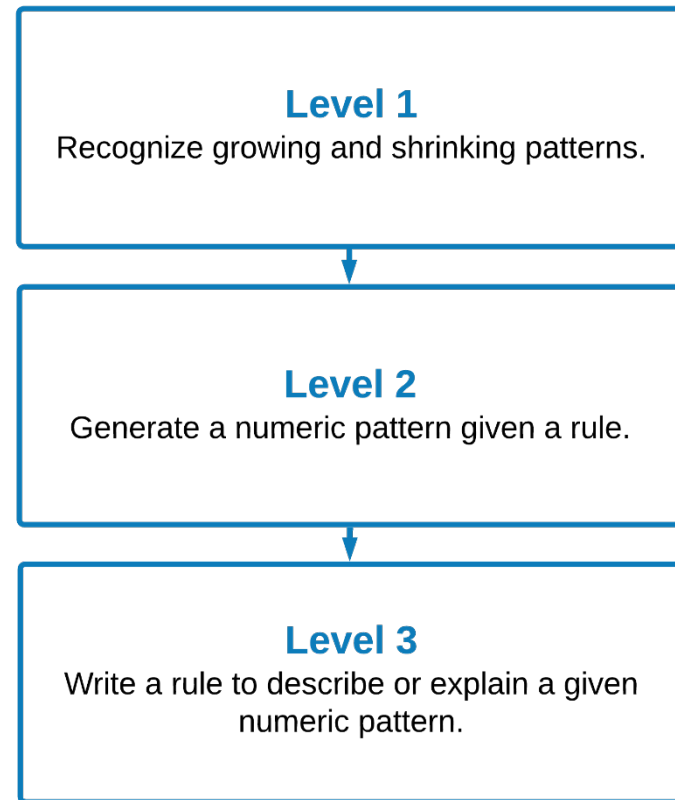## Relationships and Algebraic Thinking (RA)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.

Learning Target

5.RA.A. Represent and analyze patterns and relationships.
  2. Write a rule to describe or explain a given numeric pattern.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Recognize growing and shrinking patterns.

↓

Level 2
Generate a numeric pattern given a rule.

↓

Level 3
Write a rule to describe or explain a given numeric pattern.

PIE.5.RA.A.2 Learning Pathway Map View

PIE-F-1
Recognize same

PIE-F-2
Recognize different

PIE-F-3
Recognize attribute values

PIE-F-7
Arrange objects in pairs

PIE-F-4
Order objects

PIE-F-5
Classify

PIE-F-6
Recognize patterns in everyday context

PIE-M-35
Recognize repeating patterns

L1 PIE-M-36
Recognize growing patterns

L1 PIE-M-37
Recognize shrinking patterns

PIE-M-45
Recognize order of elements in a repeating pattern

PIE-M-38
Recognize the core unit in a repeating pattern

PIE-M-46
Organize a numeric pattern in a table

PIE-M-39
Recognize the pattern rule in a growing pattern

PIE-M-40
Recognize the pattern rule in a shrinking pattern

PIE-M-57
Explain numeric patterns

PIE-M-41
Extend a numeric pattern by applying the rule

PIE-M-93
Organize 2 distinct numeric patterns in a table

PIE-M-47
Translate a table of values into ordered pairs

L2 PIE-M-42
Generate a numeric pattern given a rule

PIE-M-43
Generate 2 numeric patterns given the rules

PIE-M-44
Translate 2 distinct numeric patterns into ordered pairs

PIE-M-56
Identify relationship between 2 numeric patterns

L3 PIE-M-58
Describe a numeric pattern with a rule

PIE-M-59
Predict a term in a numeric pattern by applying the rule

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate	
tested nodes	

PIE.5.RA.C.5
Mathematics
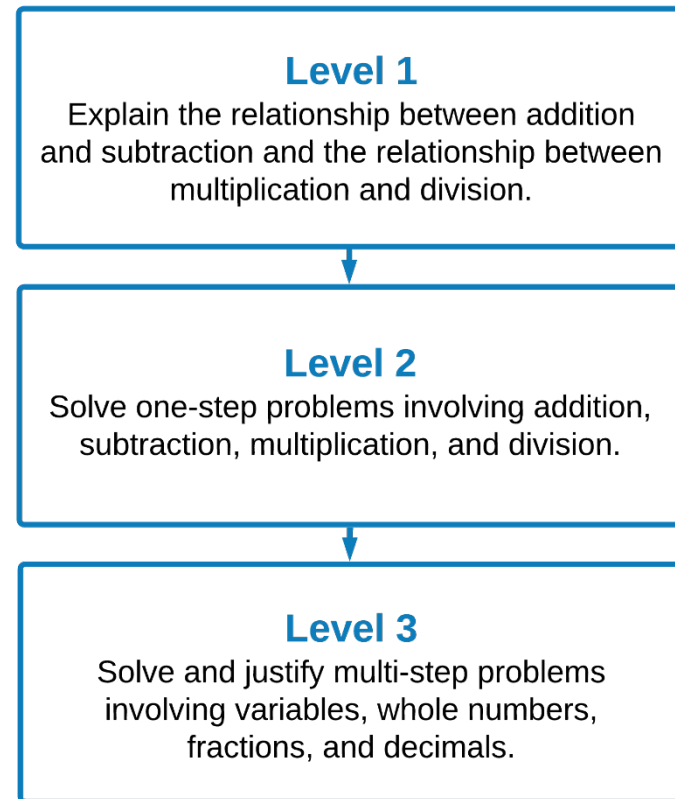## Relationships and Algebraic Thinking (RA)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
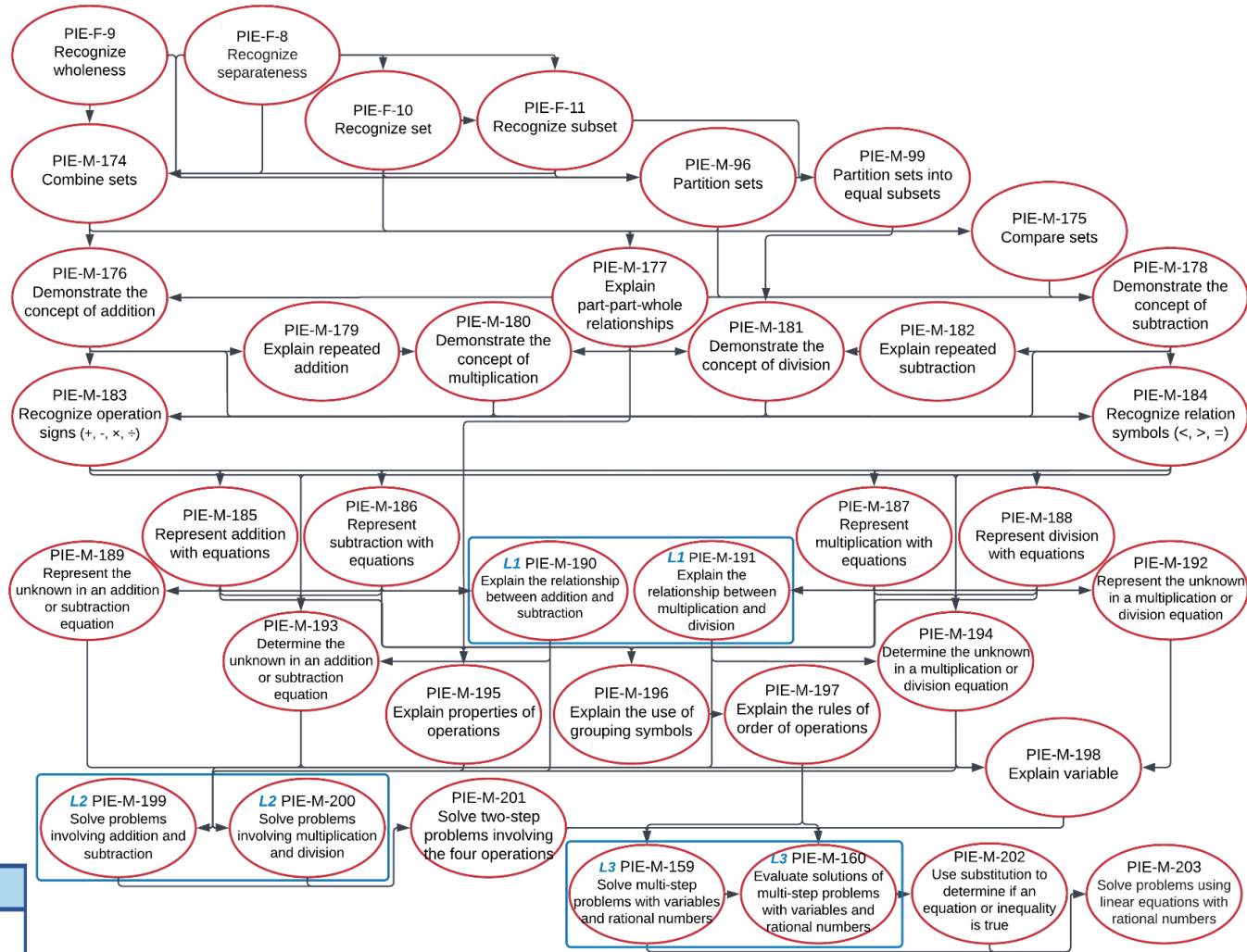
Learning Target

5.RA.C. Use the four operations to represent and solve problems.
   5. Solve and justify multi-step problems involving variables, whole numbers, fractions and decimals.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Explain the relationship between addition and subtraction and the relationship between multiplication and division.

Level 2
Solve one-step problems involving addition, subtraction, multiplication, and division.

Level 3
Solve and justify multi-step problems involving variables, whole numbers, fractions, and decimals.

PIE.5.RA.C.5 Learning Pathway Map View

Map Key

L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.GM.A.2
Mathematics
Geometry and Measurement (GM)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
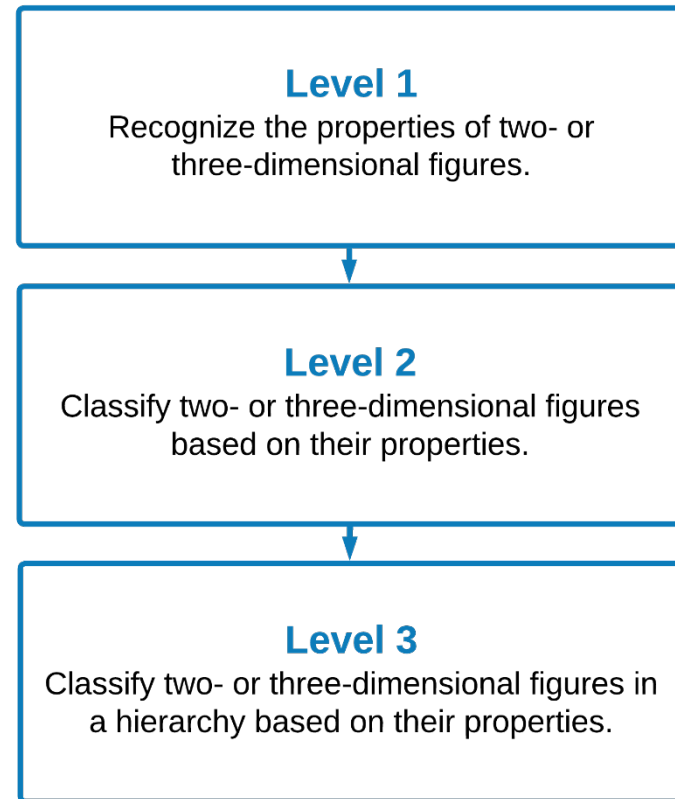
Learning Target

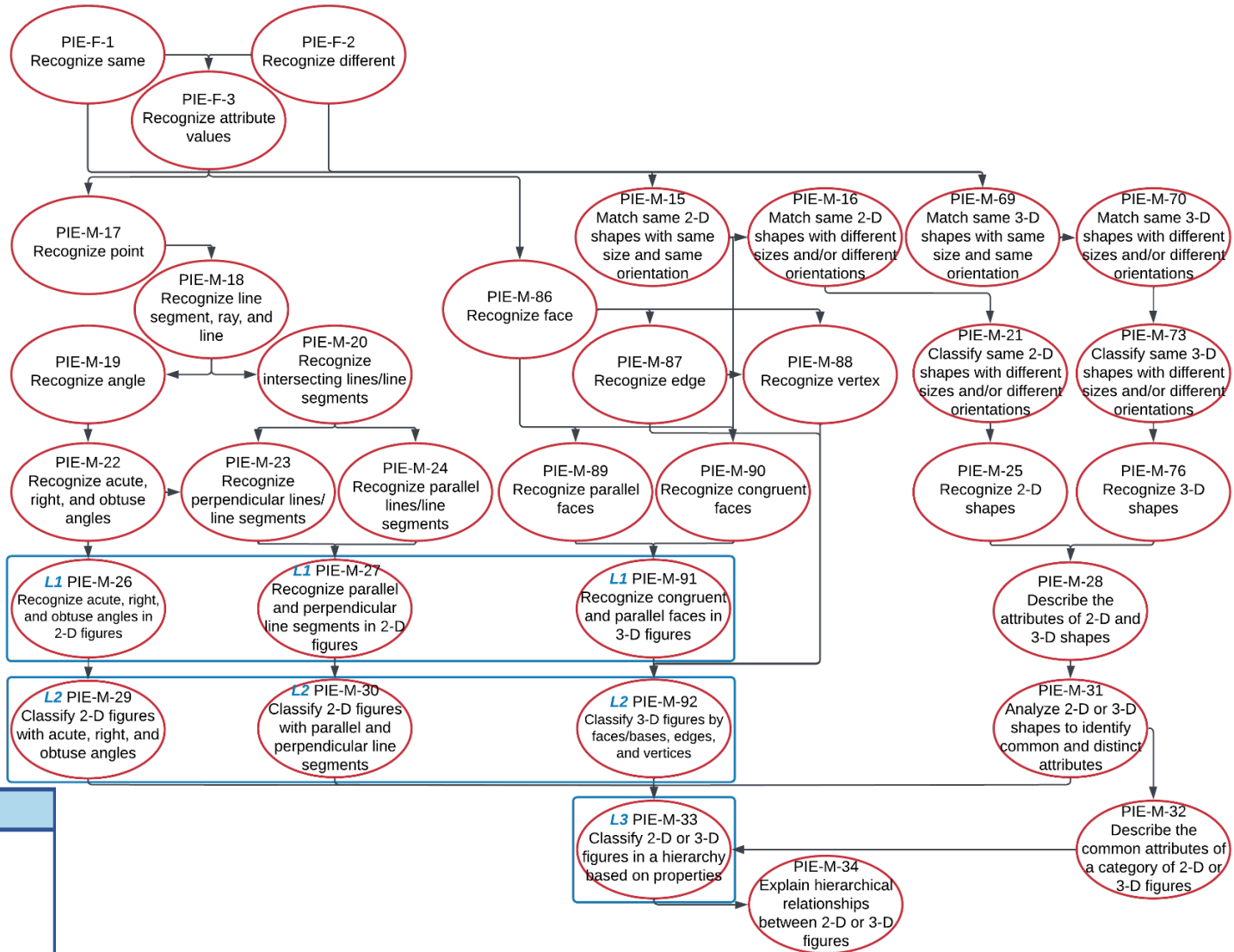**5.GM.A.** Classify two- and three-dimensional geometric shapes.
  2. Classify figures in a hierarchy based on properties.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Recognize the properties of two- or three-dimensional figures.

Level 2
Classify two- or three-dimensional figures based on their properties.

Level 3
Classify two- or three-dimensional figures in a hierarchy based on their properties.

PIE.5.GM.A.2 Learning Pathway Map View

PIE-F-1
Recognize same

PIE-F-2
Recognize different

PIE-F-3
Recognize attribute values

PIE-M-17
Recognize point

PIE-M-18
Recognize line segment, ray, and line

PIE-M-19
Recognize angle

PIE-M-20
Recognize intersecting lines/line segments

PIE-M-86
Recognize face

PIE-M-15
Match same 2-D shapes with same size and same orientation

PIE-M-16
Match same 2-D shapes with different sizes and/or different orientations

PIE-M-69
Match same 3-D shapes with same size and same orientation

PIE-M-70
Match same 3-D shapes with different sizes and/or different orientations

PIE-M-87
Recognize edge

PIE-M-88
Recognize vertex

PIE-M-21
Classify same 2-D shapes with different sizes and/or different orientations

PIE-M-73
Classify same 3-D shapes with different sizes and/or different orientations

PIE-M-22
Recognize acute, right, and obtuse angles

PIE-M-23
Recognize perpendicular lines/ line segments

PIE-M-24
Recognize parallel lines/line segments

PIE-M-89
Recognize parallel faces

PIE-M-90
Recognize congruent faces

PIE-M-25
Recognize 2-D shapes

PIE-M-76
Recognize 3-D shapes

L1 PIE-M-26
Recognize acute, right, and obtuse angles in 2-D figures

L1 PIE-M-27
Recognize parallel and perpendicular line segments in 2-D figures

L1 PIE-M-91
Recognize congruent and parallel faces in 3-D figures

PIE-M-28
Describe the attributes of 2-D and 3-D shapes

L2 PIE-M-29
Classify 2-D figures with acute, right, and obtuse angles

L2 PIE-M-30
Classify 2-D figures with parallel and perpendicular line segments

L2 PIE-M-92
Classify 3-D figures by faces/bases, edges, and vertices

PIE-M-31
Analyze 2-D or 3-D shapes to identify common and distinct attributes

L3 PIE-M-33
Classify 2-D or 3-D figures in a hierarchy based on properties

PIE-M-34
Explain hierarchical relationships between 2-D or 3-D figures

PIE-M-32
Describe the common attributes of a category of 2-D or 3-D figures

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.GM.B.4a
Mathematics
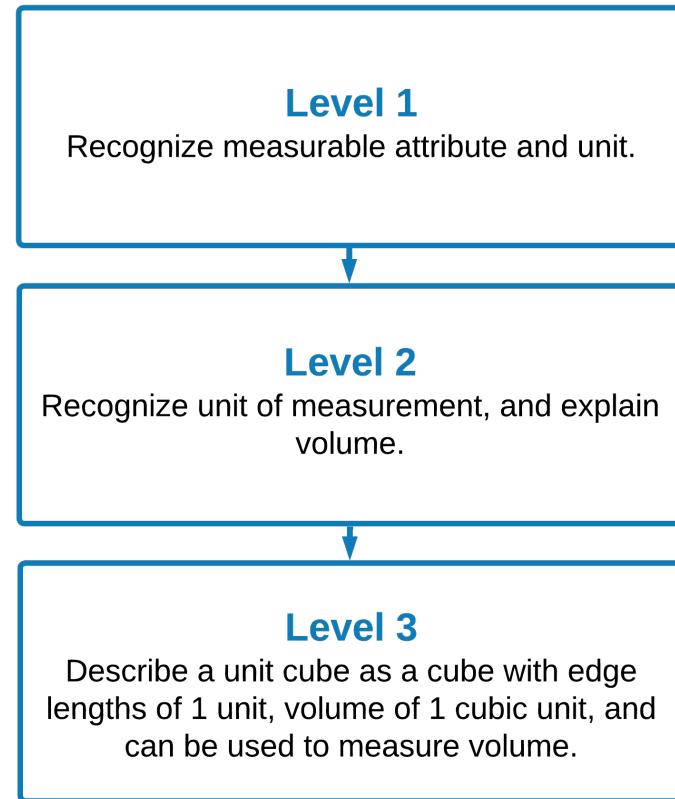## Geometry and Measurement (GM)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
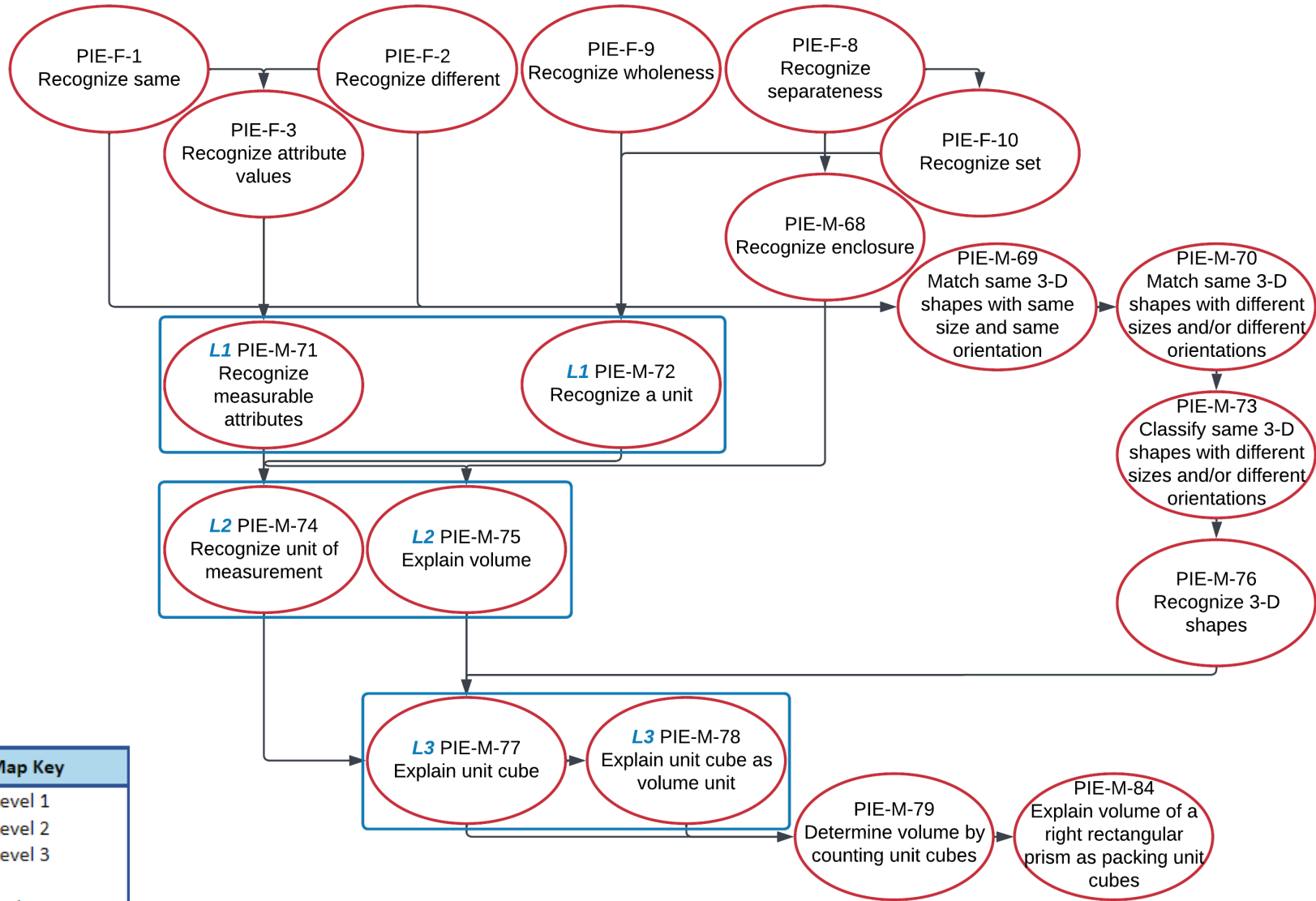
Learning Target

5.GM.B. Understand and compute volume.
   4. Understand the concept of volume and recognize that volume is measured in cubic units.
      a) Describe a cube with edge length 1 unit as a "unit cube" and is said to have "one cubic unit" of volume and can be used to measure volume.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Recognize measurable attribute and unit.

↓

Level 2
Recognize unit of measurement, and explain volume.

↓

Level 3
Describe a unit cube as a cube with edge lengths of 1 unit, volume of 1 cubic unit, and can be used to measure volume.

PIE.5.GM.B.4a Learning Pathway Map View

PIE-F-1
Recognize same

PIE-F-2
Recognize different

PIE-F-9
Recognize wholeness

PIE-F-8
Recognize separateness

PIE-F-3
Recognize attribute values

PIE-F-10
Recognize set

PIE-M-68
Recognize enclosure

PIE-M-69
Match same 3-D shapes with same size and same orientation

PIE-M-70
Match same 3-D shapes with different sizes and/or different orientations

L1 PIE-M-71
Recognize measurable attributes

L1 PIE-M-72
Recognize a unit

PIE-M-73
Classify same 3-D shapes with different sizes and/or different orientations

L2 PIE-M-74
Recognize unit of measurement

L2 PIE-M-75
Explain volume

PIE-M-76
Recognize 3-D shapes

L3 PIE-M-77
Explain unit cube

L3 PIE-M-78
Explain unit cube as volume unit

PIE-M-79
Determine volume by counting unit cubes

PIE-M-84
Explain volume of a right rectangular prism as packing unit cubes

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.GM.B.4b
Mathematics
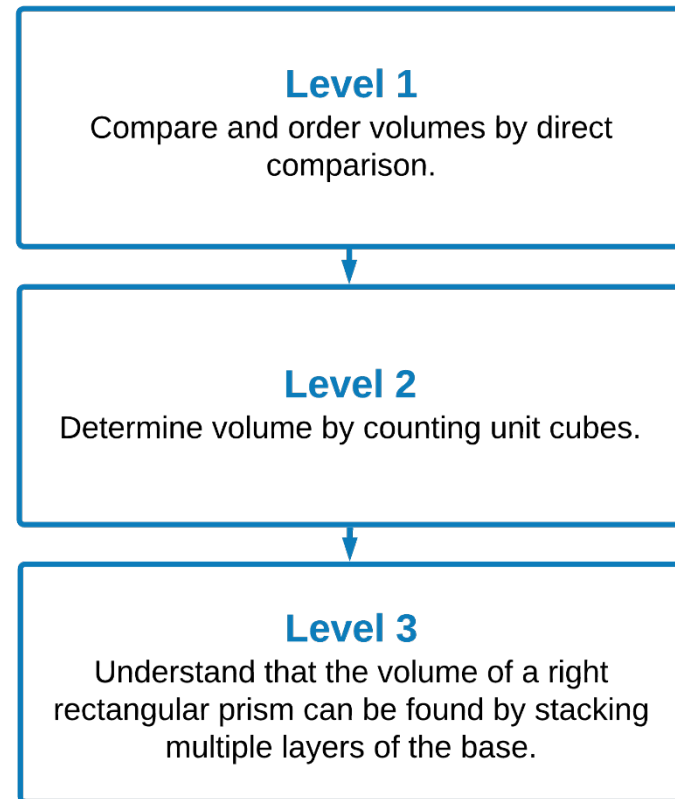## Geometry and Measurement (GM)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
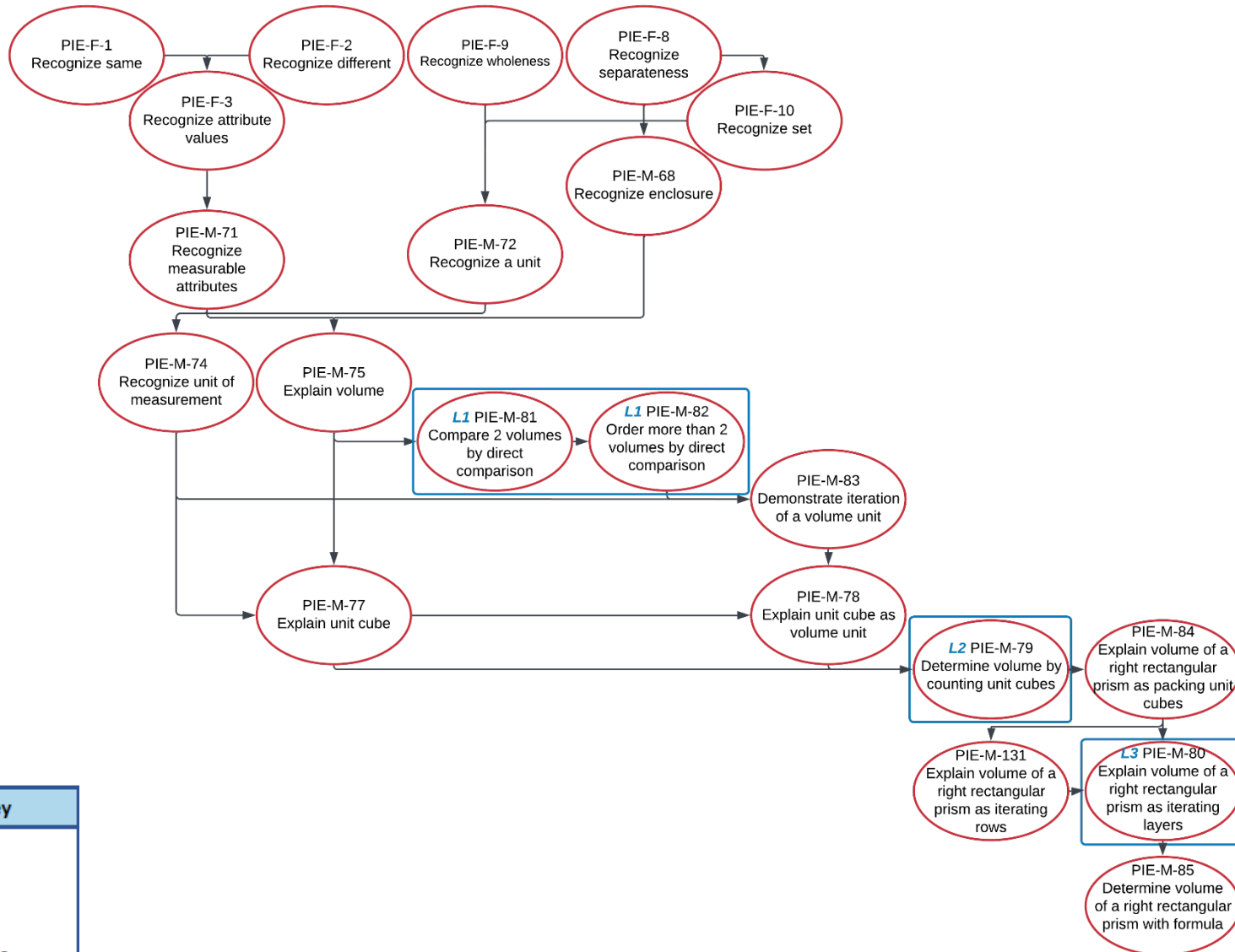
Learning Target

5.GM.B. Understand and compute volume.
  4. Understand the concept of volume and recognize that volume is measured in cubic units.
    b) Understand that the volume of a right rectangular prism can be found by stacking multiple layers of the base.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Compare and order volumes by direct comparison.

↓

Level 2
Determine volume by counting unit cubes.

↓

Level 3
Understand that the volume of a right rectangular prism can be found by stacking multiple layers of the base.

PIE.5.GM.B.4b Learning Pathway Map View

Map Key

L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes

PIE.5.GM.C.6a

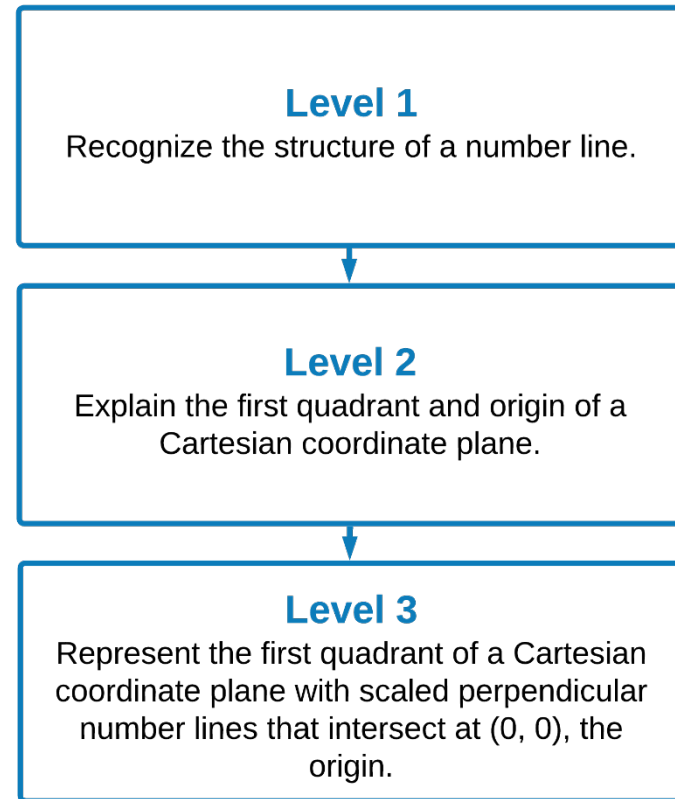Mathematics
Geometry and Measurement (GM)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
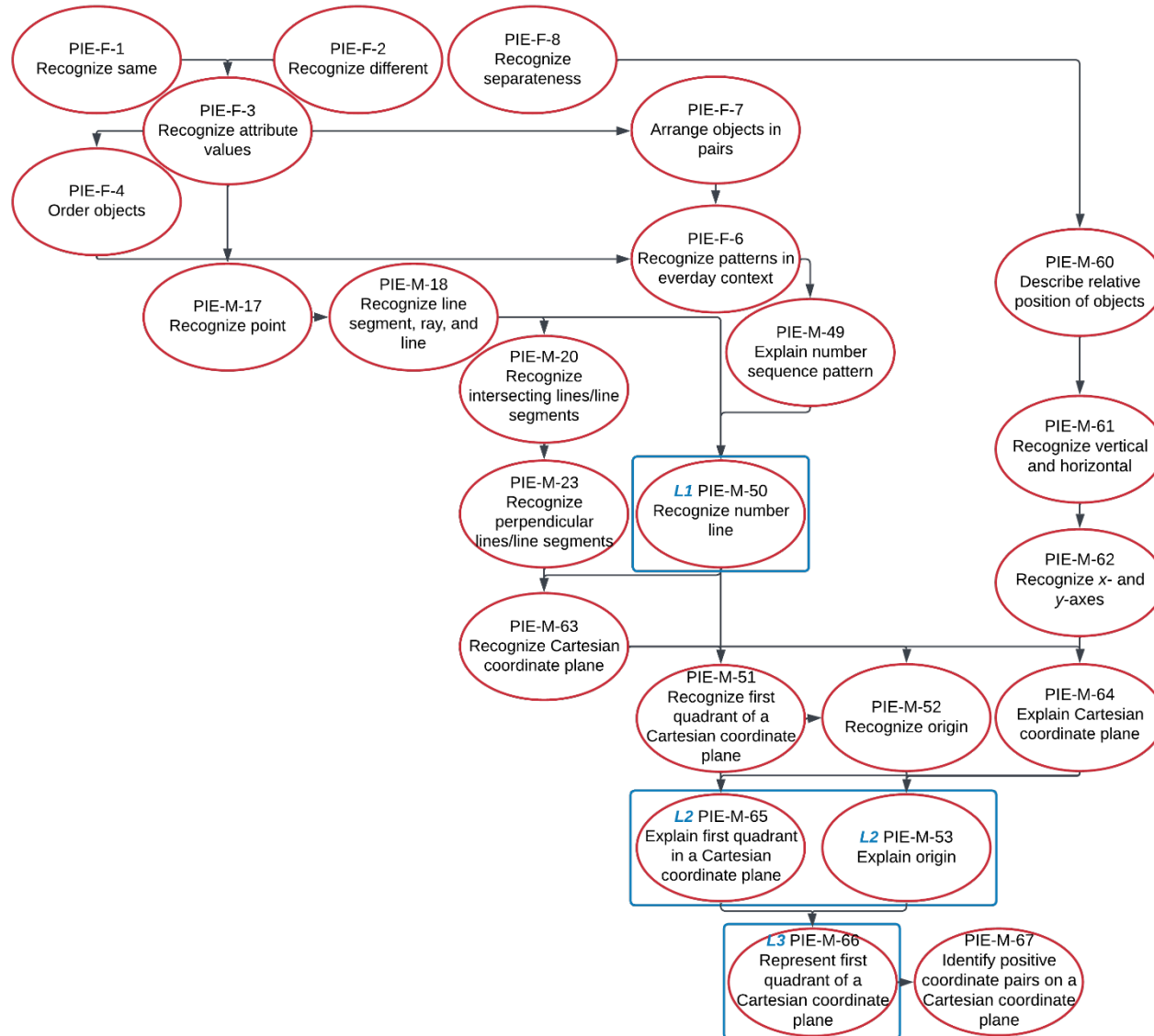
Learning Target

5.GM.C. Graph points on the Cartesian coordinate plane within the first quadrant to solve problems.
   6. Define a first quadrant Cartesian coordinate system.
      a) Represent the axes as scaled perpendicular number lines that both intersect at 0, the origin.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Recognize the structure of a number line.

↓

Level 2
Explain the first quadrant and origin of a Cartesian coordinate plane.

↓

Level 3
Represent the first quadrant of a Cartesian coordinate plane with scaled perpendicular number lines that intersect at (0, 0), the origin.

PIE.5.GM.C.6a Learning Pathway Map View

PIE-F-1
Recognize same

PIE-F-2
Recognize different

PIE-F-8
Recognize separateness

PIE-F-3
Recognize attribute values

PIE-F-7
Arrange objects in pairs

PIE-F-4
Order objects

PIE-F-6
Recognize patterns in everday context

PIE-M-60
Describe relative position of objects

PIE-M-17
Recognize point

PIE-M-18
Recognize line segment, ray, and line

PIE-M-49
Explain number sequence pattern

PIE-M-20
Recognize intersecting lines/line segments

PIE-M-61
Recognize vertical and horizontal

PIE-M-23
Recognize perpendicular lines/line segments

L1 PIE-M-50
Recognize number line

PIE-M-62
Recognize *x*- and *y*-axes

PIE-M-63
Recognize Cartesian coordinate plane

PIE-M-51
Recognize first quadrant of a Cartesian coordinate plane

PIE-M-52
Recognize origin

PIE-M-64
Explain Cartesian coordinate plane

L2 PIE-M-65
Explain first quadrant in a Cartesian coordinate plane

L2 PIE-M-53
Explain origin

L3 PIE-M-66
Represent first quadrant of a Cartesian coordinate plane

PIE-M-67
Identify positive coordinate pairs on a Cartesian coordinate plane

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.DS.A.2
Mathematics
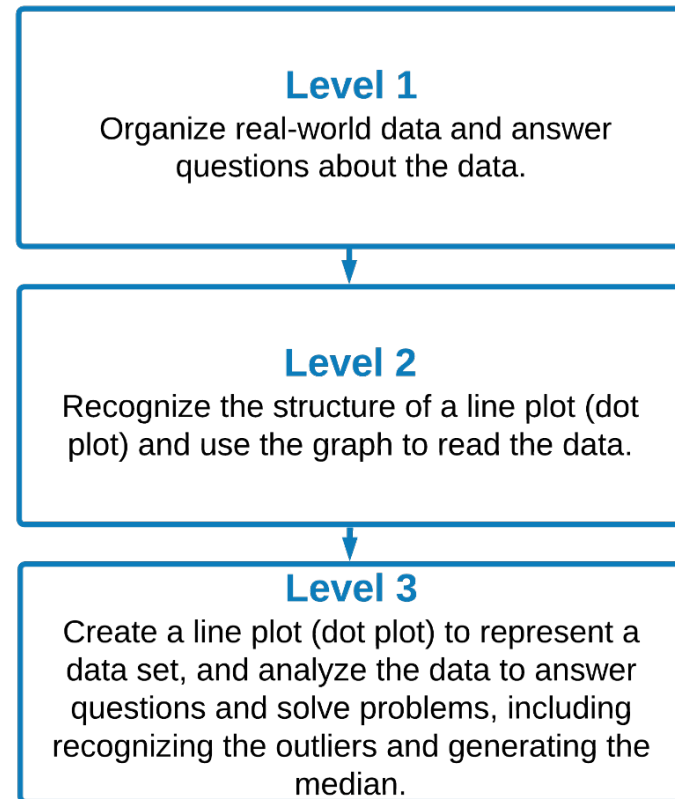Data and Statistics (DS)
Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.
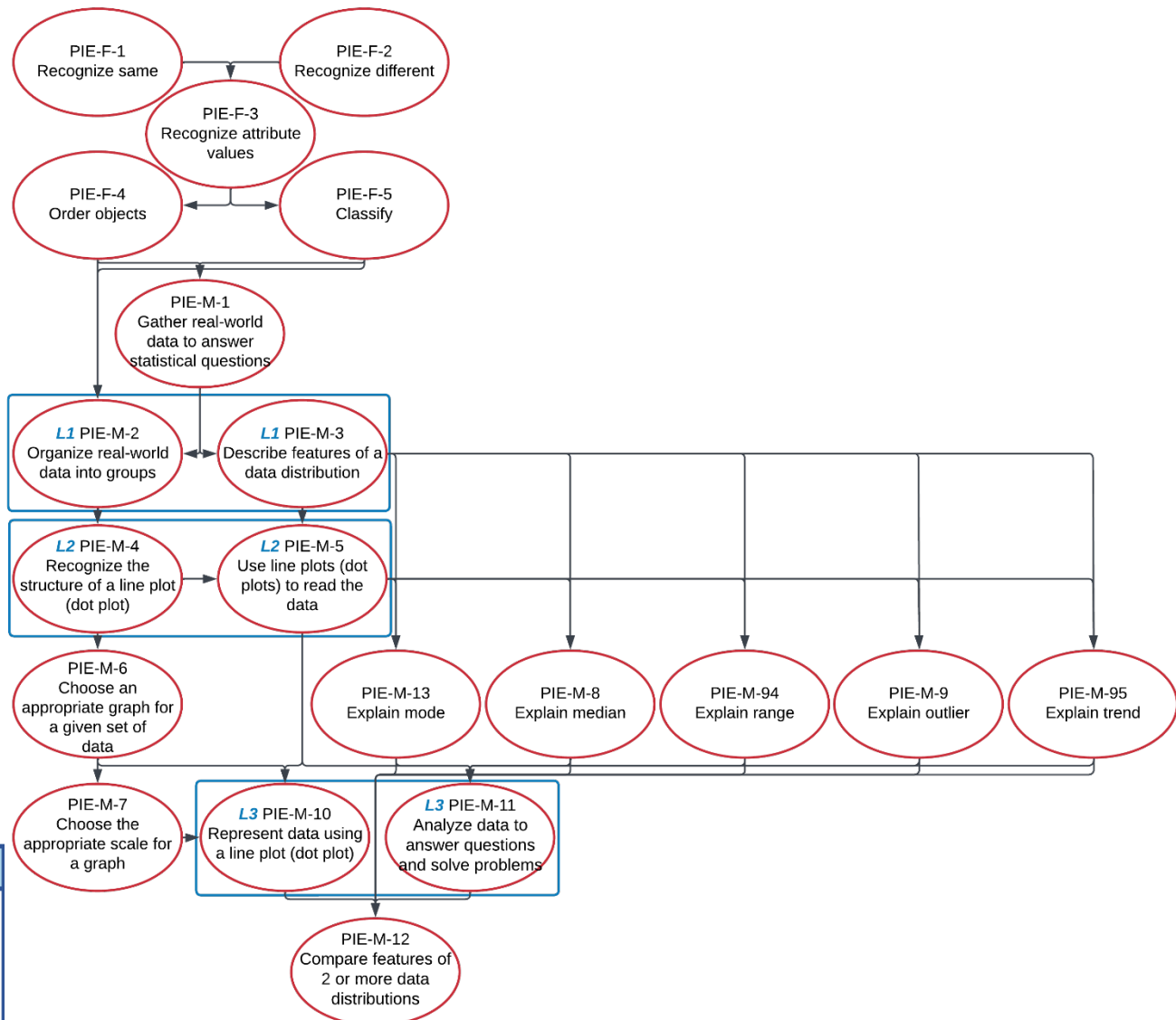
Learning Target

5.DS.A. Represent and analyze data.
   2. Create a line plot to represent a given or generated data set, and analyze the data to answer questions and solve problems, recognizing the outliers and generating the median.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.

Level 1
Organize real-world data and answer questions about the data.

↓

Level 2
Recognize the structure of a line plot (dot plot) and use the graph to read the data.

↓

Level 3
Create a line plot (dot plot) to represent a data set, and analyze the data to answer questions and solve problems, including recognizing the outliers and generating the median.

PIE.5.DS.A.2 Learning Pathway Map View

PIE-F-1
Recognize same

PIE-F-2
Recognize different

PIE-F-3
Recognize attribute values

PIE-F-4
Order objects

PIE-F-5
Classify

PIE-M-1
Gather real-world data to answer statistical questions

L1 PIE-M-2
Organize real-world data into groups

L1 PIE-M-3
Describe features of a data distribution

L2 PIE-M-4
Recognize the structure of a line plot (dot plot)

L2 PIE-M-5
Use line plots (dot plots) to read the data

PIE-M-6
Choose an appropriate graph for a given set of data

PIE-M-13
Explain mode

PIE-M-8
Explain median

PIE-M-94
Explain range

PIE-M-9
Explain outlier

PIE-M-95
Explain trend

PIE-M-7
Choose the appropriate scale for a graph

L3 PIE-M-10
Represent data using a line plot (dot plot)

L3 PIE-M-11
Analyze data to answer questions and solve problems

PIE-M-12
Compare features of 2 or more data distributions

Map Key	
L1	Level 1
L2	Level 2
L3	Level 3

Boxes indicate tested nodes