



Pathways for Instructionally Embedded Assessment

Pathways for Instructionally Embedded Assessment (PIE)
Developing Learning Pathways for the PIE Assessment
System

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Table of Contents

Introduction	1
Learning Pathways	2
Scope of Development.....	5
Learning Pathway Development.....	8
Learning Pathway Development Process	8
Preliminary Learning Pathway Development Summary.....	11
Expert Review	13
Expert Panel Recruitment and Demographics.....	13
Table Facilitators.....	14
Expert Review Process	14
Learning Pathway Review Process and Materials	17
Outcomes from the Expert Review Event.....	22
Post-Expert Review	26
Post-Expert Review Process.....	26
Results of the Post-Expert Review Process.....	27
Collected panel feedback.....	27
Panel feedback analysis and decision	30
Finalized Learning Pathway Approval.....	32
Conclusion.....	33
References	34
Appendix	36

Introduction

This technical report describes the development of learning pathways for the Pathways for Instructionally Embedded (PIE) Assessment project. The Missouri Department of Elementary and Secondary Education (DESE) leads the PIE project in partnership with the Accessible Teaching, Learning, and Assessment Systems (ATLAS) at the University of Kansas.

Funded through a Competitive Grant for State Assessments (CGSA), the PIE project aims to construct and evaluate a prototype assessment system built on cognitive models of student learning (named learning pathways) aligned with grade-level academic expectations. The prototype system includes assessments embedded throughout instruction as well as at the end of the course or year. Instructionally embedded assessments provide students the opportunity to demonstrate what they know and can do as they learn. By using learning pathways as the basis for assessments that are administered at instructionally relevant points in time, embedded assessments produce more fine-grained and timely information about student academic progress for teachers to inform their instructional decisions during the school year. The end of year assessment, in combination with the instructionally embedded assessments, supports a multiple measures approach to assessment that has potential to meet the goals of traditional spring summative assessment models. The PIE project aims to evaluate the feasibility and technical adequacy of this integrated, prototype assessment system.

A project goal for the PIE project is to design, construct, administer, and evaluate the prototype assessment system built on learning pathways aligned with grade 5 mathematics content standards. The project objective of this goal is to develop learning pathways to use as the base of assessments and reports and is the focus of this report. Regarding this project objective, the question guiding learning pathway development was: What knowledge, skills, and understandings best depict the cognitive development pathways that students follow in accomplishing grade-level academic expectations described by grade 5 mathematics content standards?

This report documents the project activities and evidence of the process taken to create the learning pathways. This report describes (a) learning pathways, (b) learning pathway development, (c) expert review, and (d) post-expert review.

The first section of this report describes the main features of learning pathways, in which three vertical levels of knowledge, skills, and understandings represent a sequence of knowledge and skill acquisition, and the scope of learning pathway development for the PIE assessment system. The second section describes the development steps taken to draft learning pathways and provides a summary of the content of the draft learning pathways. The third and fourth sections describe the expert review of the draft learning pathways in three pathway levels and the post-expert review process taken to revise and refine the draft learning pathways with pathway levels, respectively. The data generated through the expert review provide evidence of the construct of learning pathways for content and structure, as well as procedural evidence of learning pathway development.

Learning Pathways

Learning progressions refer to descriptions of “successively more sophisticated ways of reasoning within a content domain that follow one another as students learn” (Smith et al., 2006, p. 2). Learning progressions are also useful tools that can improve assessments using research on student thinking and learning (Smith et al., 2006). The PIE project proposed the construction of learning pathways as a local progression of student learning toward a particular grade-level academic expectation as a learning target, following the idea of associating academic expectations with learning progressions (Confrey et al., 2017; Confrey, 2019).

As fine-grained cognitive models, the Dynamic Learning Maps® (DLM®) learning maps (DLM Consortium, 2016) are built through an intensive research and literature synthesis process by linking nodes that represent distinct knowledge, skills, and understandings (KSUs) with unidirectional connections from less to more complex KSUs (see also Bechard et al., 2021; Kingston et al., 2016). Similar to the DLM learning maps (DLM Consortium, 2016), learning pathways depict learning targets and other significant KSUs supporting their development, including foundational cognitive skills and early academic skills, and the directional relationships between KSUs in a map view using nodes and connections. See Figure 1 for a visual representation of two nodes with one unidirectional connection in map view. In addition, learning pathways in map view delineate multiple interrelated paths students can take to master the intended learning targets by displaying connections between associated KSUs that might logically follow one another. See Figure 2 for a visual representation of multiple interrelated pathways in map view. These design features of cognitive learning model representation are a focus of the map view of learning pathways development (e.g., see Enhanced Learning Maps [ELM] learning maps, Kingston & Broaddus, 2017; Innovations in Science Map, Assessment, and Reporting Technologies [I-SMART] learning maps, Swinburne Romine et al., 2018, for the same model design features focused on in their maps).

Figure 1. A visual representation of nodes and connections in a map view. The red ovals indicate nodes and the black arrow indicates a unidirectional connection between the nodes.

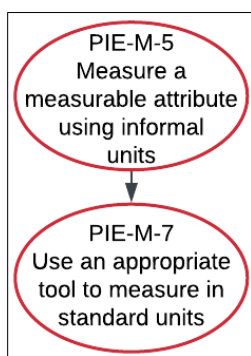
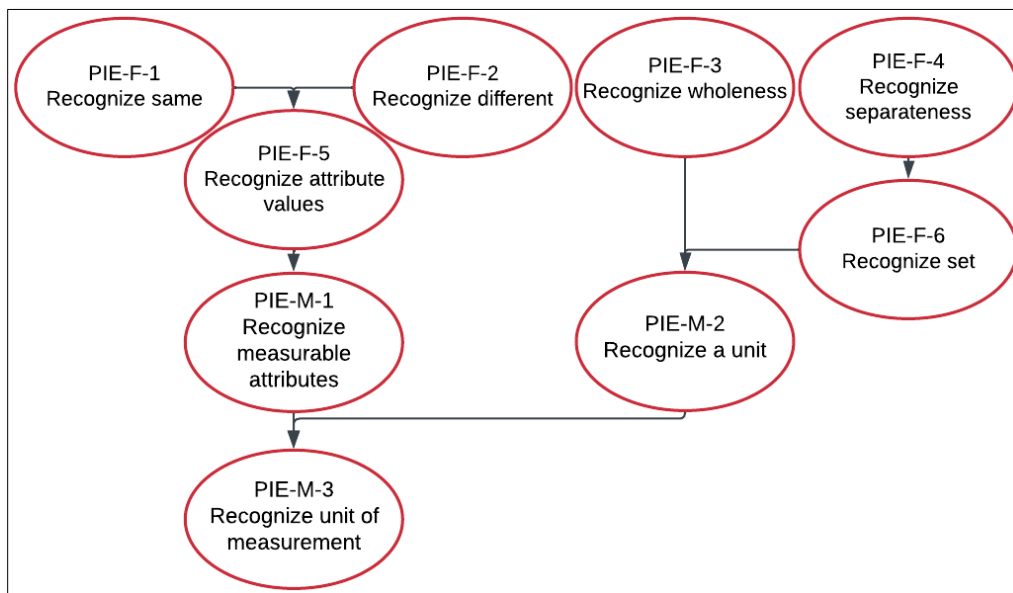


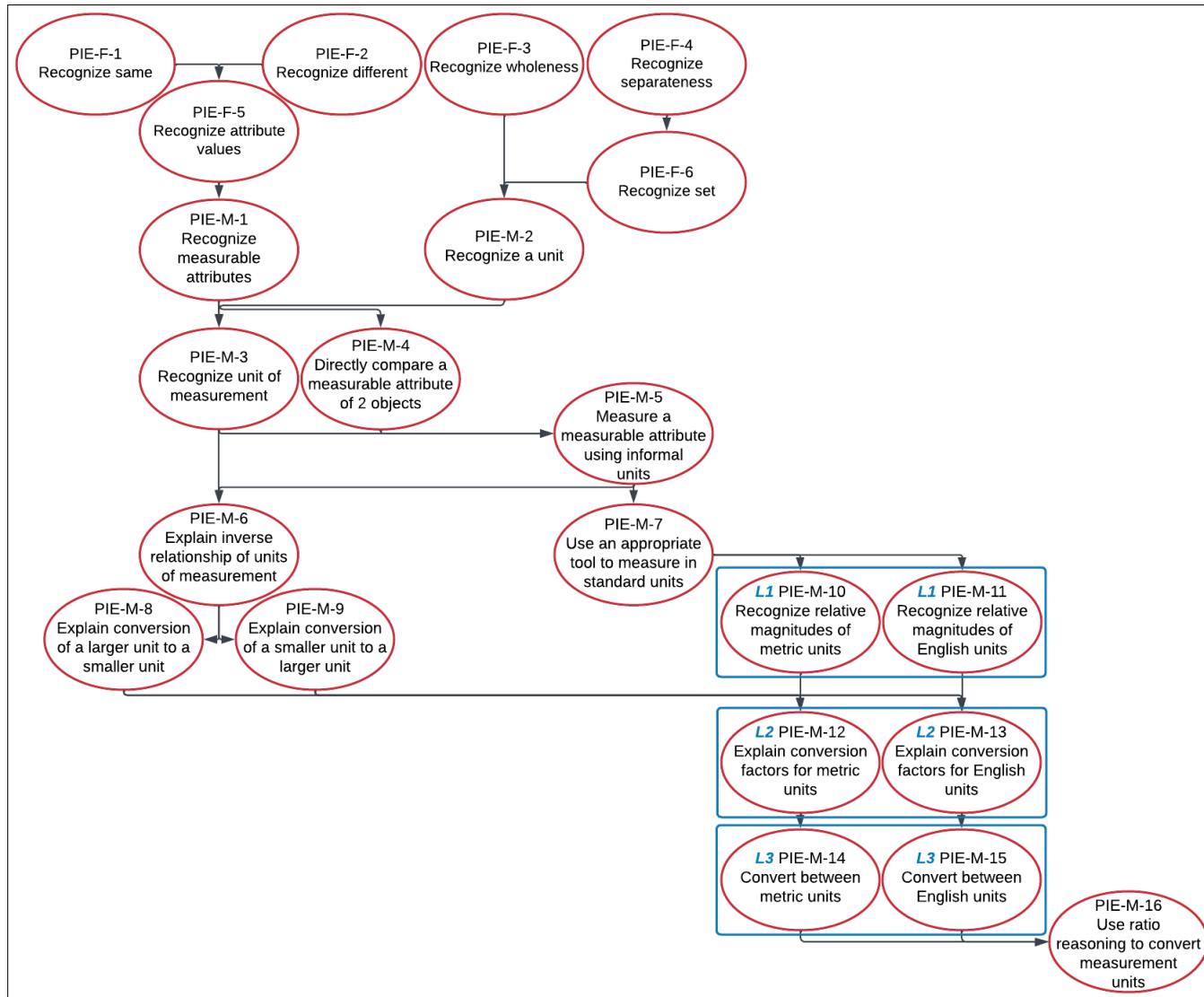
Figure 2. A visual representation of multiple interrelated paths in a map view. A node may be connected from one or more preceding nodes.



Each learning pathway depicts three vertically-related levels of distinct KSUs that are critical on the path toward learning targets that are aligned to the grade-level academic expectations, as reflected in student learning and development literature, as well as common teaching practices and curricular resources. See Figure 3 for a visual representation of learning pathways with three pathway levels in map view. The first two pathway levels (Levels 1 and 2) represent the relevant fundamental KSUs and the KSUs neighboring the learning targets of the grade-level content standards, respectively. The last level (Level 3) in the learning pathway represents the learning targets of the content standards. Each learning pathway reflects an incremental increase in cognitive complexity of KSUs from level to level. Thus, pathway levels represent a small, vertical progression to specific learning targets.¹ While the map view of learning pathways represents typical trajectories of student learning toward the content standard’s learning targets, pathway levels specify a particular point for assessments to elicit student academic progress (see Bechard et al., 2021; DLM Consortium, 2016; Kingston & Broaddus, 2017; Kingston et al., 2016; Swinburne Romine et al., 2018, for the use of linkage levels for assessment development), so educators can use this assessment data to make formative inferences about student academic progress relative to the learning pathways and plan subsequent instruction to facilitate student learning in the classroom.

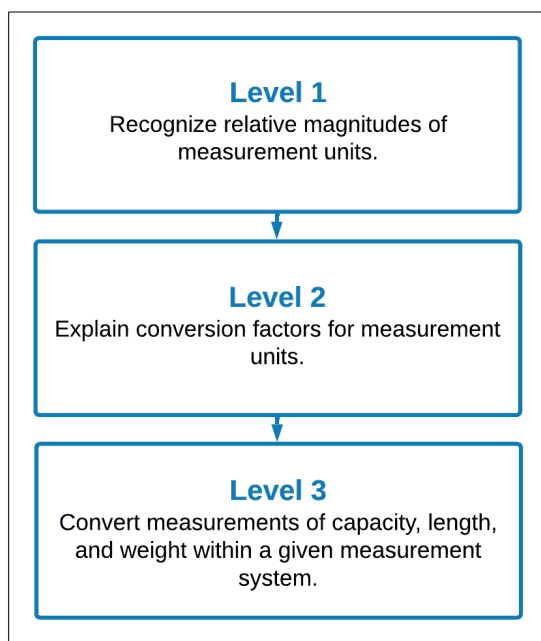
¹ Pathway levels should not be seen as corresponding in a one-to-one manner with Webb’s (2002) Depth-of-Knowledge (DOK) levels (see also Wine & Hoffman, 2023a, 2023b). Instead, student engagement with each pathway level may reflect a range of DOK levels, depending on the individual’s degree of automaticity with the KSUs therein. Therefore, it is assumed that students can access each pathway level at a range of DOK levels, depending on their current level of fluency in the skills being measured.

Figure 3. A visual representation of learning pathways with three pathway levels in a map view. The blue boxes indicate the three pathway levels. Nodes at each pathway level are marked by the level labels L1, L2, and L3 in blue.



Learning pathway level statements that describe the content of the pathway levels in the learning pathways aim to guide item development and inform the design of other system components in building the PIE assessment system. For each of the three pathway levels in the learning pathways, the learning pathway level statements summarize multiple, discrete KSUs at each pathway level from the learning pathway map view (see Bechard et al., 2021; DLM Consortium, 2016; Kingston & Broaddus, 2017; Kingston et al., 2016; Swinburne Romine et al., 2018, for the use of level descriptors for assessment development). See Figure 4 for a visual representation of three vertical pathway levels presented as learning pathway level statements.

Figure 4. A visual representation of three vertical pathway levels in learning pathway level statement. The blue boxes indicate the three pathway levels. The learning pathway level statement at each pathway level is marked by the level label Level1, Level 2, or Level 3 in blue.



Scope of Development

To prioritize learning pathway development for the PIE assessment system, the Missouri DESE selected 25 grade 5 mathematics content standards from the Priority Standards for Leveraging Learning in Mathematics (Missouri DESE, 2021) that list the mathematics content standards in grades K-12 identified as priority in considering big ideas for each grade level from the Missouri Learning Standards. Table 1 presents the full list of the 25 selected Missouri grade 5 mathematics content standards with domains and priority codes from the Priority Standards for Leveraging Learning in Mathematics (Missouri DESE, 2021).² As the domains of mathematics for grade 5, the 25 selected Missouri mathematics content standards included

- a) Number Sense and Operations in Fractions (NF; 14 standards from two clusters, 5.NF.A and 5.NF.B);

² Priority codes from the Priority Standards for Leveraging Learning in Mathematics (Missouri DESE, 2021) are used as a way of tracking the Missouri grade 5 mathematics content standards within the PIE assessment system.

- b) Relationships and Algebraic Thinking (RA; six standards from two clusters, 5.RA.A and 5.RA.C);
- c) Geometry and Measurement (GM; four standards from three clusters, 5.GM.A, 5.GM.B, and 5.GM.C); and
- d) Data and Statistics (DS; one standard from one cluster, 5.DS.A).

Table 1.

Missouri Grade 5 Mathematics Content Standards Selected for the PIE Project

Domain	Priority Code	Missouri Priority Standard
Number Sense and Operations in Fractions (NF)	5.NF.A	Understand the relationship between fractions and decimals (denominators that are factors of 100).
	5.NF.A.1	Understand that parts of a whole can be expressed as fractions and/or decimals.
	5.NF.A.2	Convert decimals to fractions and fractions to decimals.
	5.NF.A.3	Compare and order fractions and/or decimals to the thousandths place using the symbols $>$, $=$ or $<$, and justify the solution.
	5.NF.B	Perform operations and solve problems with fractions and decimals.
	5.NF.B.4	Estimate results of sums, differences and products with fractions and decimals to the thousandths.
	5.NF.B.5a	Estimate the size of the product based on the size of the two factors.
	5.NF.B.5b	Explain why multiplying a given number by a fraction greater than 1 results in a product larger than the given number.
	5.NF.B.5c	Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.
	5.NF.B.5d	Explain why multiplying the numerator and denominator by the same number is equivalent to multiplying the fraction by 1.
	5.NF.B.6	Solve problems involving addition and subtraction of fractions and mixed numbers with unlike denominators, and justify the solution.
	5.NF.B.7a	Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths.
	5.NF.B.7b	Calculate and interpret the product of a fraction by a whole number and a whole number by a fraction.

	5.NF.B.7c	Calculate and interpret the product of two fractions less than one.
	5.NF.B.8a	Calculate and interpret the quotient of a unit fraction by a non-zero whole number.
	5.NF.B.8b	Calculate and interpret the quotient of a whole number by a unit fraction.
Relationships and Algebraic Thinking (RA)	5.RA.A	Represent and analyze patterns and relationships.
	5.RA.A.1a	Generate two numeric patterns given two rules.
	5.RA.A.1b	Translate two numeric patterns into two sets of ordered pairs.
	5.RA.A.1c	Graph numeric patterns on the Cartesian coordinate plane.
	5.RA.A.1d	Identify the relationship between two numeric patterns.
	5.RA.A.2	Write a rule to describe or explain a given numeric pattern.
	5.RA.C	Use the four operations to represent and solve problems.
	5.RA.C.5	Solve and justify multi-step problems involving variables, whole numbers, fractions and decimals.
Geometry and Measurement (GM)	5.GM.A	Classify two- and three- dimensional geometric shapes.
	5.GM.A.2	Classify figures in a hierarchy based on properties.
	5.GM.B	Understand and compute volume.
	5.GM.B.4a	Describe a cube with edge length 1 unit as a “unit cube” and is said to have “one cubic unit” of volume and can be used to measure volume.
	5.GM.B.4b	Understand that the volume of a right rectangular prism can be found by stacking multiple layers of the base.
	5.GM.C	Graph points on the Cartesian coordinate plane within the first quadrant to solve problems.
	5.GM.C.6a	Represent the axes as scaled perpendicular number lines that both intersect at 0, the origin.
Data and Statistics (DS)	5.DS.A	Represent and analyze data.
	5.DS.A.2	Create a line plot to represent a given or generated data set, and analyze the data to answer questions and solve problems, recognizing the outliers and generating the median.

Note. Missouri Priority Standards are from Missouri DESE (2021). Bold indicates cluster (priority code and description) as a heading linking the standards to their domain (Missouri DESE, 2021).

PIE project staff developed learning pathways aligned to the 25 selected Missouri grade 5 mathematics content standards taken as learning targets. Table 2 presents the learning pathway development coverage of Missouri grade 5 mathematics priority standards from the Priority Standards for Leveraging Learning in Mathematics (Missouri DESE, 2021). The learning pathway development scope for the PIE assessment system covered 25 (86.2%) of the 29 *Missouri* grade 5 mathematics priority standards across four domains. Selection of the standards for PIE learning pathway development facilitated construction of the learning pathways on key ideas, concepts, and skills focused on by the Missouri mathematics content standards at the grade 5 level.

Table 2.

Learning Pathway Development Coverage of Missouri Grade 5 Mathematics Priority Standards

Domain	Cluster	Missouri Priority Standards	Missouri Priority Standards Selected for PIE Development
Number Sense and Operations in Fractions (NF)	5.NF.A	3	3
	5.NF.B	11	11
Relationships and Algebraic Thinking (RA)	5.RA.A	5	5
	5.RA.C	1	1
	5.GM.A	1	1
Geometry and Measurement (GM)	5.GM.B	2	2
	5.GM.C	4	1
	5.GM.D	1	0
	5.DS.A	1	1
Total		29	25

Note. Missouri Priority Standards are from Missouri DESE (2021).

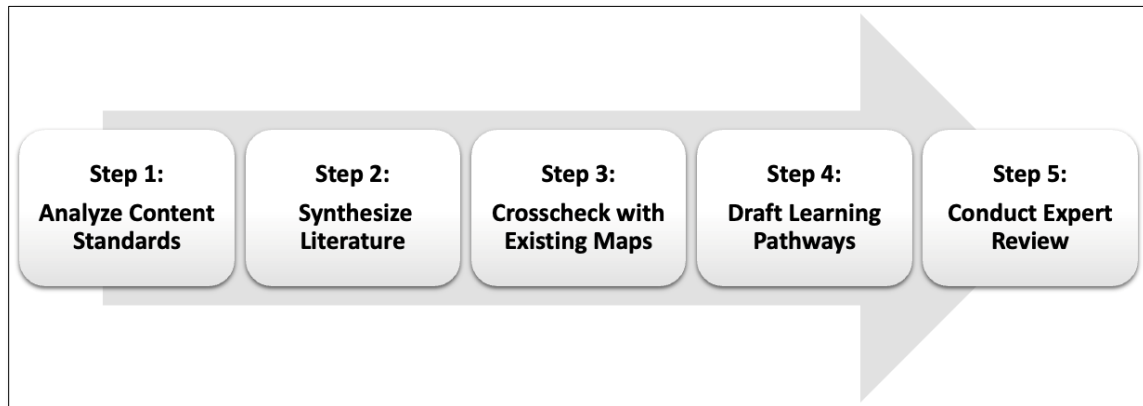
Learning Pathway Development

The PIE project proposed to construct learning pathways with three pathway levels by adapting the established learning map development process (Bechard et al., 2021; DLM Consortium, 2016; Swinburne Romine et al., 2018). Learning pathways represented in the map views provide the basis for the pathway levels. This section presents the learning pathway development process.

Learning Pathway Development Process

Using adaptations of the established learning map development process (Bechard et al., 2021; DLM Consortium, 2016; Swinburne Romine et al., 2018), the process used to develop the learning pathways included five main steps, as shown in Figure 5.

Figure 5. An overview of the learning pathway development process




The first step of this learning pathway development process was to analyze the 25 selected Missouri grade 5 mathematics content standards from the Priority Standards for Leveraging Learning in Mathematics (Missouri DESE, 2021) to identify target KSUs or learning targets. The identified learning targets for the content standard were represented as individual target nodes in the learning pathways. Some content standards with multiple learning targets were represented as separate target nodes, reflecting their distinct KSUs.

The second step of the learning pathway development process was to synthesize available literature on student learning and development to describe a typical trajectory of student learning toward the identified learning targets, as well as precursors (foundational cognitive skills and early academic skills) to student learning. Literature types synthesized at this stage included existing mathematics learning progressions and learning trajectories, empirical research, and literature reviews. The information from common teaching practices and curricular resources was also synthesized. This literature synthesis process included

- a) searching the relevant literature for learning pathways by using concepts and skills addressed in the content standards as literature search keywords and selecting the literature for teaching and learning the key concepts and skills in early childhood, elementary, and middle school mathematics education;
- b) identifying precursors and critical KSUs that build toward and comprise the learning targets represented by the content standards from the literature; and
- c) organizing the precursors and critical KSUs into several different clusters (preliminary, being at each level, transitional between levels, and succeeding), related to grade bands and grade levels, to construct the learning pathways.

Research briefs summarized the literature synthesized in constructing learning pathways to support the audience's understanding of the precursors and critical KSUs represented in the learning pathways. See Figure 6 for an example of a research brief.

Figure 6. An example of a research brief



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RESEARCH BRIEF

Learning Pathway: 5.NF.A.3

LEARNING TARGET CONTENT STANDARD

5.NF.A.3 Understand the relationship between fractions and decimals (denominators that are factors of 100).
3. Compare and order fractions and/or decimals to the thousandths place using the symbols $>$, $=$, or $<$, and justify the solution.

OVERVIEW

Learning pathway for 5.NF.A.3 (PIE.5.NF.A.3 learning pathway) represents multiple learning paths students may take to acquire target knowledge, skills, and understandings of the grade-level content standard 5.NF.A.3 in (a) comparing fractions and/or decimals up to thousandths using symbols, (b) ordering fractions and/or decimals up to thousandths using symbols, and (c) evaluating results of fraction and/or decimal comparisons. PIE.5.NF.A.3 learning pathway displays key knowledge, skills, and understandings (31 nodes) and their relationships (64 connections).

LEARNING PATHWAY RESEARCH BACKGROUND

Students' work with fraction ordering reveals their understanding of the quantities the fractions represent (Cramer et al., 2008). In this PIE.5.NF.A.3 learning pathway, there are two distinct but closely associated paths of student learning: one for fraction comparison and the other for decimal comparison.

According to Clements & Sarama (2021), children in the 7 to 8 age range show their learning and development in fraction comparison from comparing fraction models and stating which one is the larger number (7 years), to comparing fractions with small denominators using physical fraction models, to comparing fractions with small denominators using fraction models, such as number lines (8 years).

Building on their previous learning of length comparison with a standard measurement unit in grade 2, third-graders learn to compare fractions with like denominators (Common Core Standards Writing Team [CCSWT], 2018). Third-graders can see the fractions with the same denominator as having the foundational unit fractions of the same size, so they determine the fraction with the lesser numerator is less because it is made of fewer unit fractions (CCSWT, 2018). As with equivalent fractions, it is essential in fraction comparison that both fractions state the same whole (CCSWT, 2018). In comparing unit fractions, third-graders can see the unit fraction with the lesser denominator is greater, thinking that when needing a smaller number of equal-size parts to make the same whole, the parts must be greater (CCSWT, 2018; e.g., Cramer et al., 2008).

Understanding equivalent fractions supports comparison of fractions with unlike numerators and denominators (CCSWT, 2018). Fourth-graders learn to compare fractions with unlike numerators and denominators using benchmarks, such as $\frac{1}{2}$ and 1 (CCSWT, 2018). Following their learning and development in adding and subtracting simple common fractions with physical fraction models and multiplying and dividing simple common fractions with rectangular array models, children (8 years) learn to order positive fractions (Clements & Sarama, 2021). As third-graders see fractions as points on a number line, they learn about the idea of order as position

(CCSWT, 2018). For instance, in comparison of two fractions as two points on a number line, the fraction to the right is greater than the other fraction (CCSWT, 2018).

Decimal point specifies "the location of the ones (or units) place" by sitting to the right of the units position (Van de Walle et al., 2019, p. 407). Decimals like 0.45 can be read as 4 tenths and 5 hundredths, linking to fraction numeration (Van de Walle et al., 2019). Reading decimals using place-value labels supports students to see the relationship between fraction notation and decimal notation, like when hearing three tenths, thinking both 0.3 and $\frac{3}{10}$ (Van de Walle et al., 2019). Reading decimals using place-value labels also supports students to connect the base-ten system to decimals and compare decimals, like comparing 0.50 and 0.5 (Loehr & Rittle-Johnson, 2016; see also Van de Walle et al. 2019). Fourth-graders learn to write decimal fractions (referring to fractions with factors of 100) in decimal notation (CCSWT, 2018). Students can think of the equivalence $1.90 = 1.9$, related to fraction conversion $1.90 = \frac{190}{100} = (\frac{19 \times 10}{10 \times 10}) = \frac{19}{10} = 1.9$ (CCSWT, 2018), or using a 10×10 grid (Cramer et al., 2009). Comparing decimal fractions and placing the decimal fractions in order is closely related to fraction comparison and decimal comparison (Van de Walle et al., 2019).

Like decimal equivalence, students compare decimals in terms of seeing decimals as fractions (CCSWT, 2018). For instance, students may compare 0.3 and 0.07, thinking of them as $\frac{3}{10}$ and $\frac{7}{100}$ and seeing $\frac{3}{10} > \frac{7}{100}$ (CCSWT, 2018). In grade 5, area models support student work with decimals up to hundredths (CCSWT, 2018; e.g., Cramer et al., 2009). The idea of decimals as fractions can support student work with decimals beyond hundredths (CCSWT, 2018). Children (8 years) learn to order positive decimals (Clements & Sarama, 2021; see also Cramer et al., 2009, 2015). Activities using a 10×10 grid can support students to build the idea of order for decimals (Cramer et al., 2009, 2015). Student learning of fraction comparison further progresses to estimating the results of fraction addition and subtraction (e.g., Cramer et al., 2008) and the results of decimal addition and subtraction (e.g., Cramer et al., 2015).

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PIE.5.NF.A.3

Page 1 of 2

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PIE.5.NF.A.3

Page 2 of 2

The third step of the learning pathway development process was to crosscheck the construction of the learning pathways built on the literature synthesis with other existing learning maps from the DLM Alternate Assessment System and the Enhanced Learning Maps (ELM) project, ensuring a similar progression relative to the intended learning targets.

The fourth step of the learning pathway development process was to draft learning pathways and indicate their pathway levels by identifying level content appropriately sized and distinct from adjacent levels and representing a logical learning sequence toward the learning targets (Swinburne Romine et al., 2018), aiming to represent appropriate and accessible learning paths for the majority of students. This step of the learning pathway development process included drafting learning pathway level statements using common practices. The common practices followed by PIE project staff in drafting learning pathway level statements included

- a) where possible, generating a simple and unique level statement;
- b) beginning the sentence with a verb (e.g., recognize, explain, convert) as assessment targets;
- c) using a general and overarching level statement for multiple, discrete KSUs; and
- d) ensuring the Level 3 statement is aligned to the content standard’s learning target(s), only with some language addition or removal for clarity and cohesiveness with its preceding learning pathway level statements.

The draft learning pathways with pathway levels in map view and their draft learning pathway level statements were reviewed by internal experts (ATLAS staff) with expertise in learning map or assessment development.

The construction of learning maps built on available literature is typically validated through an expert review process and empirical evaluation research (Clark & Karvonen, 2021; see also DLM Consortium, 2016). Like learning maps, the fifth step of the learning pathway development process was to conduct an expert review to gather panel feedback on the draft learning pathways in three pathway levels.

Preliminary Learning Pathway Development Summary

In response to the guiding question of what KSUs best depict the cognitive development pathways that students follow in accomplishing grade-level academic expectations described by grade 5 mathematics content standards, the learning pathway development process produced 25 draft learning pathways aligned with the selected Missouri grade 5 mathematics content standards from the Priority Standards for Leveraging Learning in Mathematics (Missouri DESE, 2021). The draft learning pathways consisted of, in total, 213 nodes, 476 connections, and 75 pathway levels (three levels per learning pathway). Table 3 presents the node and connection information for each learning pathway.

Table 3.

Primary Learning Pathway Content Before Expert Review

Learning Pathway	Node	Connection
PIE.5.NF.A.1	24	44

PIE.5.NF.A.2	25	46
PIE.5.NF.A.3	31	64
PIE.5.NF.B.4	33	52
PIE.5.NF.B.5a	25	43
PIE.5.NF.B.5b	26	42
PIE.5.NF.B.5c	28	44
PIE.5.NF.B.5d	26	43
PIE.5.NF.B.6	29	58
PIE.5.NF.B.7a	28	47
PIE.5.NF.B.7b	33	58
PIE.5.NF.B.7c	33	57
PIE.5.NF.B.8a	31	50
PIE.5.NF.B.8b	29	44
PIE.5.RA.A.1a	20	35
PIE.5.RA.A.1b	22	39
PIE.5.RA.A.1c	24	36
PIE.5.RA.A.1d	24	48
PIE.5.RA.A.2	25	49
PIE.5.RA.C.5	38	91
PIE.5.GM.A.2	34	48
PIE.5.GM.B.4a	19	23
PIE.5.GM.B.4b	21	26
PIE.5.GM.C.6a	24	31
PIE.5.DS.A.2	20	41
Total	213	476

Expert Review

ATLAS partnered with the Missouri DESE to host an on-site expert review event in June 2023. The purpose of this event was to gather feedback from educators on the draft learning pathways in three pathway levels using their experience and expertise. This section presents the findings of this expert review event, which is the last step of the learning pathways development process.

The review process for the draft learning pathways aimed to evaluate their content and structure using expert feedback from a panel of educators. Additionally, the learning pathway review process was intended to gain procedural evidence of the learning pathway development.

Expert Panel Recruitment and Demographics

PIE project staff identified the desired panelist qualifications and panel size for the expert review event. The desired panelist qualifications used to recruit the expert panel included teaching or content expertise in third- to sixth-grade mathematics. The expert panel recruitment process aimed to recruit enough panelists for two panels, with four panelists in each.

In the first step of the recruitment process, the Missouri DESE distributed a recruitment message to school districts and elementary and middle schools to recruit educators interested in participating in the expert review. In addition, PIE project staff sent the recruitment message to qualified educators in an existing database of educators who met the desired panelist qualifications for the expert review and who had indicated interest in participating in ATLAS events. PIE project staff also nominated educators within ATLAS with the appropriate experience and expertise for this expert review.

The second step of the expert panel recruitment process included sending an email with a link to a voluntary expert review screener survey to potential panelists, who notified PIE project staff of their participation interest. The screener survey collected information from the educators on multiple topics, including demographics and experience. PIE project staff used the potential panelists' survey responses to ensure the recruitment of only qualified panelists for the expert review. The expert panel recruitment process recruited eight panelists to participate in the expert review event, meeting the desired panel size for the expert review.

Table 4 presents information on the recruited panelists' years of teaching experience in K-12 and mathematics. According to their responses to the screener survey, recruited panelists had between eight and 26 years of teaching experience in K-12, with a median of 20.5 years. For mathematics teaching experience, recruited panelists had between two and 26 years, with a median of 15.5 years.

Table 4.

Panelists' Years of Teaching Experience (N = 8)

	Range	Median
K-12	8-26	20.5
Mathematics	2-26	15.5

Of the eight recruited panelists, four panelists hold a master's degree, and the other four panelists hold other advance degrees, specified by them as "Ed.D.," "Specialist," "Administration and Educational Technology," and "Currently completing Specialists in Educational Admin[i]stration," respectively.

Table 5 presents the information on the recruited panelists' profession. As their professional role or job title, two panelists specified a teacher, one panelist specified a teacher and instructional coach, one panelist specified a teacher, LEA (district) staff, and instructional coach, one panelist specified a teacher, LEA (district) staff, instructional coach, and community member, one panelist specified an LEA (district) staff, and two panelists specified an item writer or developer. The recruited panelists showed significant experience and expertise appropriate to serve for this expert review.

Table 5.

Panelists' Profession (N= 8)

Role	<i>n</i>	%
Teacher	2	25.0
Teacher and instructional coach	1	12.5
Teacher, LEA (district) staff, and instructional coach	1	12.5
Teacher, LEA (district) staff, instructional coach, and community member	1	12.5
LEA (district) staff	1	12.5
Item writer/developer	2	25.0

Table Facilitators

Each panel at the expert review had a table facilitator (PIE project staff) to support the panel review process, including facilitating panel review and discussions and recording panel feedback. Both table facilitators had prior experience in facilitating panel review processes at other ATLAS projects' events for learning map or assessment development.

Prior to the on-site expert review event, table facilitators completed an hour-long facilitator training, including learning pathway review criteria. This facilitator training provided

- a) event overview and goals;
- b) event information (days, times, and location), event agenda, and on-site event logistics with brief descriptions of roles and responsibilities;
- c) information of event materials, including panelist binders, facilitator binders, table binders, project SharePoint folders, and laptops;
- d) event walk-through;
- e) learning pathway review process and procedures, including review criteria; and
- f) review process walk-through.

Expert Review Process

The expert review process encompassed an online advance training session and on-site training and expert review event. Before attending the on-site expert review event, panelists received an advance training module to prepare them for participation in the expert review. As a prerecorded presentation, this 21-minute-long training module introduced

- a) project overview and goals;
- b) learning pathways;
- c) expert review purpose and learning pathway review process; and
- d) event information (days, times, and location).

The on-site expert review event was held over two days in Columbia, Missouri on June 1-2, 2023. The first day of the event began with an orientation session that recapitulated all the main points addressed in the online advance training. Following this orientation session, panelists received an on-site training that provided more details on the learning pathway review process, including learning pathway review criteria, and engaged in a practice review activity to become more familiar with the learning pathway review process and procedures, as well as given review materials and resources (e.g., binders, rating sheets, node books). Table 6 presents the learning pathway review criteria used at the expert review. The learning pathway review criteria were generated using adaptations of the existing learning map development and review criteria (DLM Consortium, 2016; see also DLM Science Consortium, 2017; Swinburne Romine et al., 2018). Table 7 presents the learning pathways assigned to each panel. Panel formation for the expert review intended to assign 11 learning pathways from the domains of Relationships and Algebraic Thinking, Geometry and Measurement, and Data and Statistics to panel 1 and 14 learning pathways from the domain of Number Sense and Operations in Fractions to panel 2, balancing between the number of learning pathways and the conceptual association between learning pathways to be reviewed during the expert review.

Table 6.

Pathway Level Content and Structure Review

Panel Feedback		Review Criterion	Evaluation
Content	Alignment	The pathway level has a direct or clear relation with a content standard when it represents either: <ul style="list-style-type: none"> • KSUs that are addressed in the content standard’s learning target(s) • KSUs that support the development of the content standard’s learning target(s) 	For each pathway level, assess as having either: <ul style="list-style-type: none"> • partial or no alignment • complete or clear alignment
	Size	The pathway level is appropriate when it represents: <ul style="list-style-type: none"> • appropriately sized KSUs • KSUs distinct from the preceding and/or succeeding pathway levels 	For each pathway level, assess as being either: <ul style="list-style-type: none"> • inappropriate or redundant • appropriate and distinct
	Clarity	The pathway level is clear when it uses: <ul style="list-style-type: none"> • clear language to describe KSUs • correct math vocabulary to describe KSUs 	For each pathway level, assess as being either: <ul style="list-style-type: none"> • unclear or incorrect • clear and correct

Structure	Consistency	<p>The pathway levels of a learning pathway cover all the KSUs represented by individual nodes at the corresponding pathway levels when the pathway levels' KSUs:</p> <ul style="list-style-type: none"> are consistent with the KSUs represented by the nodes at the pathway levels 	<p>For each pathway level, assess as having either:</p> <ul style="list-style-type: none"> inconsistent consistent
	Progression	<p>The pathway levels of a learning pathway reflect a logical progression to the content standard's learning target(s) when each pathway level's KSUs:</p> <ul style="list-style-type: none"> reflects an incremental cognitive complexity as a short, vertical progression to the content standard's specific learning target(s) are necessary and sufficient for progressing toward the KSUs at the succeeding pathway level 	<p>For each pathway level, assess as having either:</p> <ul style="list-style-type: none"> illogical or no progression logical progression

Table 7.

Learning Pathway Assignments by Panel

Panel	Domain	Learning Pathway
Panel 1	Relationships and Algebraic Thinking (RA)	PIE.5.RA.A.1a
		PIE.5.RA.A.1b
		PIE.5.RA.A.1c
		PIE.5.RA.A.1d
		PIE.5.RA.A.2
		PIE.5.RA.C.5
	Geometry and Measurement (GM)	PIE.5.GM.A.2
		PIE.5.GM.B.4a
		PIE.5.GM.B.4b
		PIE.5.GM.C.6a
Panel 2	Data and Statistics (DS)	PIE.5.DS.A.2
	Number Sense and Operations in Fractions (NF)	PIE.5.NF.A.1
		PIE.5.NF.A.2
		PIE.5.NF.A.3
		PIE.5.NF.B.4
		PIE.5.NF.B.5a
		PIE.5.NF.B.5b
		PIE.5.NF.B.5c

	PIE.5.NF.B.5d
	PIE.5.NF.B.6
	PIE.5.NF.B.7a
	PIE.5.NF.B.7b
	PIE.5.NF.B.7c
	PIE.5.NF.B.8a
	PIE.5.NF.B.8b
Total	25

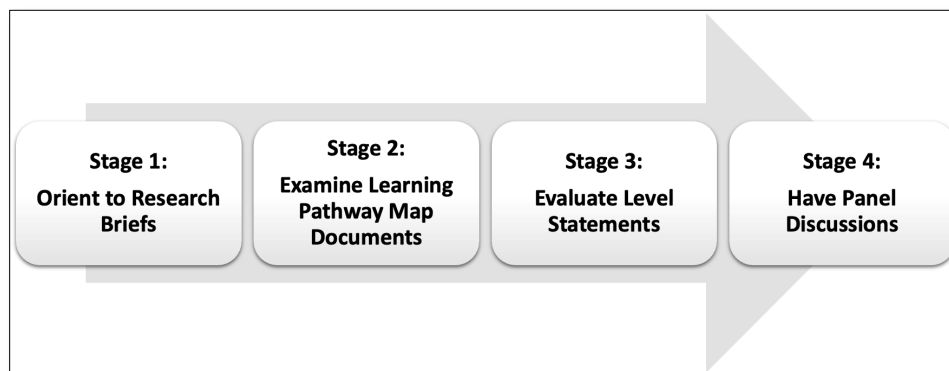
Next, during the panel review session activity, the panelists independently evaluated the pathway levels in their assigned learning pathways against the learning pathway review criteria and then discussed their review with the group to produce the panel's rating and feedback on the pathway levels. At the first day's wrap-up session, the panelists had an opportunity to ask questions about the first day review.

On the second day of the expert review event, the panelists continued reviewing the pathway levels in their assigned learning pathways, following the same review process as the first day. At the conclusion of the expert review event, panelists completed a panelist feedback survey and received travel reimbursement and compensation information.

Learning Pathway Review Process and Materials

At the on-site expert review event, panelists evaluated pathway levels of draft learning pathways on their content and structure, following established procedures used for learning maps review (Bechard et al., 2021; DLM Consortium, 2016; Swinburne Romine et al., 2018). Panelists used a four-stage learning pathway review process to review learning pathways in three pathway levels, as shown in Figure 7.

Figure 7. An overview of the learning pathway review process



The first stage of the learning pathway review process was to read the research briefs about learning pathways that were provided in panelist binders. The second stage of the learning pathway review process was to examine learning pathway map documents that were also provided in panelist binders. Through these first two stages of the learning pathway review process, panelists became familiar with the content and structure of the draft learning pathways and their three pathway levels to be evaluated at the expert review event. The third stage of the

learning pathway review process was to make individual ratings of pathway levels of assigned learning pathways for each learning pathway review criterion and record their ratings and feedback on individual rating sheets accessed via the project SharePoint. Types of feedback provided by panelists included

- a) recommendations for specific change(s) to the pathway levels not meeting the review criteria (revision recommendations); and
- b) additional suggestions to refine the pathway levels meeting the review criteria (refinement suggestions).

After completing individual ratings, as the fourth stage of this learning pathway review process, panelists had a group discussion to make consensus ratings on the pathway levels of their assigned learning pathways for each review criterion and developed panel feedback (revision recommendations or refinement suggestions provided by the expert panel). Table facilitators supported group discussions and recorded panel ratings and feedback on panel rating sheets in the project SharePoint.

Review materials and resources used at the on-site expert review event included binders and rating sheets. Types of binders used at the expert review were

- a) panelist binders that consisted of a confidentiality agreement document, an event agenda, event day 1 PowerPoint slides, a review process document, a practice activity learning pathway map document, and research briefs and learning pathway map documents of assigned learning pathways;
- b) facilitator binders that consisted of an event agenda, event day 1 PowerPoint slides, a review process document, a practice activity learning pathway map document, research briefs and learning pathway map documents of assigned learning pathways, and a table facilitator manual; and
- c) table binders of node books.

Research briefs provided a short summary of the existing literature about student learning and the development of KSUs represented in each learning pathway to help panelists understand critical KSUs for learning pathways. As a three-page-long document for each learning pathway, learning pathway map documents presented well-organized information about the learning pathway. See Appendix on page 36 for an example of each page of a practice activity learning pathway map document. The first page of the learning pathway map document presents the learning pathway overview. The second page presents descriptions and observations of KSUs for each of the three pathway levels. The third page of the learning pathway map document presents a learning pathway map view. Rating sheets were accessed via the project SharePoint. Types of rating sheets used at the on-site expert review were

- a) individual rating sheets accessed by the panelists; and
- b) panel rating sheets accessed by the table facilitators.

Both rating sheets had the same content and structure. Each rating sheet in a spreadsheet consisted of

- a) a spreadsheet tab for the information of learning pathway review criteria and a review progress tracker;
- b) spreadsheet tabs for content and structure review of learning pathways in pathway levels;
- c) a spreadsheet tab for node book; and
- d) a spreadsheet tab for general feedback.

See Figure 8 and Figure 9 for examples of the individual rating sheets used during the panelists' content review and structure review of the learning pathways in pathway levels. The node book tab in the spreadsheet presents the content of the nodes displayed in the map view of learning pathways. Spreadsheet node book and table binder node book had the same content and structure.

Figure 8. An example of individual rating sheets—Content review

Learning Pathway	Learning Target	Level	Level Statement	Level Nodes	Content Criterion 1: Alignment	Content Criterion 1 Comments	Content Criterion 2: Size	Content Criterion 2 Comments	Content Criterion 3: Clarity	Content Criterion 3 Comments
PIE.5.GM.D.8	5.GM.D.8. Convert measurements of capacity, length, and weight within a given measurement system.	3	Convert measurements of capacity, length, and weight within a given measurement system.	<div>PIE-M-14 Convert between metric units</div> <div>PIE-M-15 Convert between English units</div>	<div>complete or clear alignment</div> <div>partial or no alignment</div>					
		2	Explain conversion factors for measurement units.	<div>PIE-M-12 Explain conversion factors for metric units</div> <div>PIE-M-13 Explain conversion factors for English units</div>						
		1	Recognize relative magnitudes of measurement units.	<div>PIE-M-10 Recognize relative magnitudes of metric units</div> <div>PIE-M-11 Recognize relative magnitudes of English units</div>						

Figure 9. An example of individual rating sheets–Structure review

Learning Pathway	Learning Target	Level	Level Statement	Level Nodes	Structure Criterion 1: Consistency	Structure Criterion 1: Comments	Structure Criterion 2: Progression	Structure Criterion 2: Comments
PIE.5.GM.D.8	5.GM.D.8. Convert measurements of capacity, length, and weight within a given measurement system.	3	Convert measurements of capacity, length, and weight within a given measurement system.	PIE-M-14 Convert between metric units	<div>consistent</div> <div>inconsistent</div>	<div>▼</div>		
				PIE-M-15 Convert between English units				
		2	Explain conversion factors for measurement units.	PIE-M-12 Explain conversion factors for metric units				
				PIE-M-13 Explain conversion factors for English units				
		1	Recognize relative magnitudes of measurement units.	PIE-M-10 Recognize relative magnitudes of metric units				
PIE-M-11 Recognize relative magnitudes of English units								

Outcomes from the Expert Review Event

Following the on-site expert review event, panelists completed a panelist feedback survey. Survey responses provided data about the learning pathway review process and resources in terms of

- a) the effectiveness of training components;
- b) the importance of advance and on-site training and review materials;
- c) the quality of the review process at each stage; and
- d) their overall expert review event experience.

Table 8 presents information on the panelists' perceived effectiveness of training components in helping to review learning pathways. According to the survey responses, panelists perceived all the training components as being either somewhat effective or very effective.

Table 8.

Panelists' Perceived Effectiveness of Training (N = 8)

	Not At All Effective		Somewhat Effective		Very Effective		Not Applicable	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Advance training	0	0	3	37.5	5	62.5	0	0
On-site training	0	0	3	37.5	5	62.5	0	0
On-site practice activity	0	0	1	12.5	7	87.5	0	0
On-site resource materials	0	0	1	12.5	7	87.5	0	0
Discussion with other panelists	0	0	0	0	8	100	0	0
Guidance from the table facilitator	0	0	1	12.5	7	87.5	0	0
Guidance from PIE project staff subject matter experts	0	0	1	12.5	7	87.5	0	0

Table 9 presents information on the panelists' perceived importance of advance and on-site training and review materials in preparing as a panelist at the expert review. Across the five review resources, panelists perceived the on-site practice activity as being the most important one, while perceiving the advance training as being the least important one. For instance, a panelist explained her rating of the on-site practice activity as being the most important one: "The practice activity really helped me feel more confident with the review process. The scaffolded approach with being able to get lots of support from staff was extremely helpful." Another panelist explained her rating of the advance training as being the least important one: "I wasn't able to ask questions when I didn't understand."

Table 9.

Panelists' Perceived Importance of Training and Materials (N = 8)

	First Place		Second Place		Third Place		Fourth Place		Fifth Place		Total Ranking
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	
Advance training	0	0	1	12.5	1	12.5	3	37.5	3	37.5	5th
On-site review of resource materials	1	12.5	3	37.5	2	25.0	1	12.5	1	12.5	3rd
On-site training: Review of advance training	1	12.5	1	12.5	0	0	3	37.5	3	37.5	4th
On-site training: Review criteria	1	12.5	3	37.5	3	37.5	0	0	1	12.5	2nd
On-site practice activity	5	62.5	0	0	2	25.0	1	12.5	0	0	1st

Table 10 presents information on the panelists' perceived overall quality of the learning pathway review process at each stage. Most of the panelists were either satisfied or very satisfied with the perceived quality of the learning pathway review process at each stage. One panelist who was dissatisfied with Stage 2 described the barriers or difficulties she encountered during the learning pathway review process: "The vocabulary was new and was not explained well. Also, the process really needed to be walked through as a whole group step by step rather than a quick overview."

Table 10.

Panelists' Perceived Quality of Learning Pathway Review Process at Each Stage (N = 8)

	Very Dissatisfied		Dissatisfied		Neither Dissatisfied or Satisfied		Satisfied		Very Satisfied	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Stage 1: Orient to research briefs	0	0	0	0	1	12.5	2	25.0	5	62.5
Stage 2: Examine learning pathway map documents	0	0	1	12.5	0	0	1	12.5	6	75.0
Stage 3: Evaluate level statements	0	0	0	0	1	12.5	2	25.0	5	62.5
Stage 4: Have panel discussions	0	0	0	0	0	0	1	12.5	7	87.5

Table 11 presents information on the panelists' perceived overall expert review event experience. Most panelists agreed or strongly agreed with the confidence statements about their overall expert review event experience. In particular, all the panelists strongly agreed with

the confidence statement, “My panel provided constructive feedback when making recommendations about the PIE learning pathways.”

Table 11.

Expert Review Panelists’ Overall Experience (N = 8)

	Strongly Disagree		Disagree		Neither Disagree or Agree		Agree		Strongly Agree	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
The overall goals and objectives for the Expert Review were clear.	0	0	0	0	0	0	4	50.0	4	50.0
I had enough time to independently make my review ratings.	0	0	0	0	1	12.5	4	50.0	3	37.5
I am confident that I applied the review criteria for my review appropriately.	0	0	0	0	1	12.5	4	50.0	3	37.5
My panel provided constructive feedback when making recommendations about the PIE learning pathways.	0	0	0	0	0	0	0	0	8	100
The table facilitator was effective at guiding panelists through the review process and panel discussion.	0	0	0	0	0	0	2	25.0	6	75.0
PIE project staff were knowledgeable about learning pathway content.	0	0	0	0	0	0	2	25.0	6	75.0
Overall, I valued the PIE learning pathway review process as a professional development experience.	0	0	0	0	0	0	2	25.0	6	75.0

The survey results provided evidence of the usefulness and quality of the learning pathway review process and presented review materials and resources. Survey responses (e.g., In the

online advance training that was a prerecorded presentation, panelists were not able to ask and have questions answered) provided information on how to better plan the PIE project's further assessment development events, as well as other ATLAS projects' events.

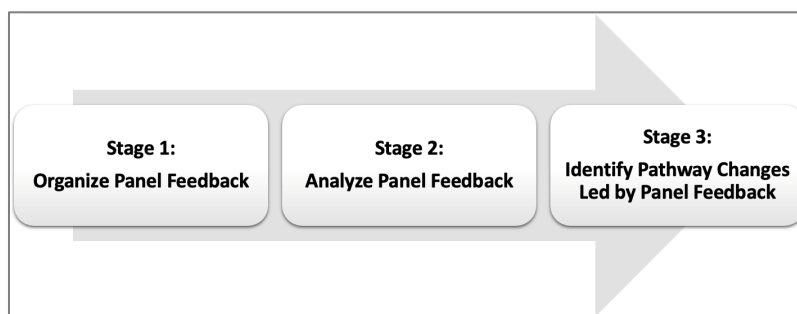
Post-Expert Review

After the on-site expert review event, PIE project staff collaborated with Missouri DESE staff on a post-expert review process. The purpose of the post-expert review process was to revise and refine the draft learning pathways in three pathway levels using the panel feedback collected from the expert review event. This section presents the post-expert review process and results.

Post-Expert Review Process

Following an established process of panel feedback evaluation (Swinburne Romine et al., 2018), PIE project staff reviewed all the collected panel feedback (revision recommendations for specific change[s] to the pathway levels not meeting the review criteria or refinement suggestions despite meeting the review criteria) and accepted the panel feedback based on panel feedback review criteria. PIE project staff used a three-stage post-expert review process to evaluate the collected panel feedback, as shown in Figure 10.

Figure 10. An overview of the post-expert review process



The first stage of the post-expert review process was to organize the collected panel feedback in a spreadsheet with tabs for the panel feedback provided on the panel rating sheet against the learning pathway review criteria. The spreadsheet tabs also included columns where PIE project staff document their analysis and decision at the following stages.

The second stage of the post-expert review process was to analyze the collected panel feedback based on panel feedback review criteria for soundness and consistency, using a panel feedback evaluation process document and the evaluation sheet organized at the first stage of the post-expert review process. The panel feedback review criteria were generated by adapting the existing criteria used for collected panel feedback evaluation in the learning map post-panel review process (Swinburne Romine et al., 2018). The panel feedback review criteria used by PIE project staff to analyze the panel feedback included

- a) ensuring that the panel feedback is based on valid reasoning and sound judgment to satisfy the content accuracy and relevance of pathway level (soundness); and
- b) ensuring that the panel feedback complies with the essential characteristics of pathway level to satisfy the structural coherence of the learning pathway (consistency).

During this second stage of the post-expert review process, a PIE project staff member first assessed the applicability of panel feedback against the panel feedback review criteria and recorded their evaluation ratings (accept or reject) and explanations of the ratings on the

evaluation sheet. Another PIE project staff member then reviewed the evaluation ratings made by the first staff member and recorded their evaluation ratings (agree or disagree) and explanations of the ratings on the evaluation sheet. Both PIE project staff members who reviewed the collected panel feedback had expertise in mathematics content and assessment development. After the individual panel feedback evaluation, PIE project staff had a group discussion to make consensus decisions on whether to accept or reject the panel feedback based on the panel feedback review criteria and then record the final evaluation ratings and their explanations for the ratings on the evaluating sheets.

The third stage of the post-expert review process was to identify specific changes that need to be made to the pathway levels as led by the accepted panel feedback. PIE project staff indicated specific change(s) for the pathway level with one or more of the listed change codes representing their change types. The change codes were generated by adapting the existing coding scheme used for accepted panel feedback categorization in learning map panel feedback evaluation (Swinburne Romine et al., 2018). Table 12 presents the different types of changes that were indicated for pathway levels.

Table 12.

Pathway Level Indicated Change

Change Type		Change Description
Content	Alignment	Pathway level modification for the sake of better alignment with the content standard's learning target(s).
	Size	Pathway level modification for the sake of appropriate level size and distinct from the preceding and/or succeeding levels.
	Clarity	Pathway level content edit for the sake of clear or consistent language.
Structure	Transfer	Node transfer from the current pathway level to the other pathway level for the sake of structural coherence or vertical articulation.
	Inclusion	Node inclusion to a pathway level for the sake of structural coherence or vertical articulation.
	Exclusion	Node exclusion from the current pathway level for the sake of structural coherence or vertical articulation.

Results of the Post-Expert Review Process

At the expert review event, extensive panel feedback was generated and collected. Expert panels made consensus ratings and provided panel feedback (revision recommendations and refinement suggestions) on the draft learning pathways in three pathway levels against the learning pathway review criteria for content (alignment, size, and clarity) and structure (consistency and progression).

Collected panel feedback

The panel feedback organization stage of the post-expert review process produced the panel feedback counts for the content and structure of the draft learning pathways by counting the numbers of pathway levels with panel feedback. Table 13 summarizes the amount of collected

panel feedback on the content and structure of the draft learning pathways in three pathway levels. In total, 108 panel feedback were collected, regarding the content review criteria for alignment, size, and clarity, and structure review criteria for consistency and progression.

Across 25 draft learning pathways, the expert panel provided panel feedback on 25 (33.3%), 12 (16.0%), and 35 (46.7%) of 75 pathway levels' content based on alignment, size, and clarity, respectively. Most of the pathway level content feedback called for changes to improve the clarity of language. For instance, for the draft learning pathway level statement of "Recognize the structure of a line plot (dot plot) and use the graph to read the data," the expert panel flagged this learning pathway level statement for its clarity and recommended to "Rewrite to 'Identify the parts of a line plot and use the graph to answer questions about the data.' Rationale: [S]ome teachers might not be familiar or know what 'structures' mean."

Across 25 draft learning pathways, the expert panel provided feedback on seven (9.3%) and 29 (38.7%) of 75 pathway levels' structure based on consistency and progression, respectively. Most of the pathway level structure feedback called for changes to improve vertical progression. For instance, for the draft learning pathway level statement of "Explain fraction multiplication," the expert panel flagged this learning pathway level statement: "Seems to be more challenging with the algorithm than it needs be at level 2."

Table 13.

Panel Feedback Summary—Counts of Collected Panel Feedback on Pathway Level Content and Structure by Review Criterion and Learning Pathway

Learning Pathway	Collected Panel Feedback on Pathway Level Content			Collected Panel Feedback on Pathway Level Structure	
	Alignment Criterion	Size Criterion	Clarity Criterion	Consistency Criterion	Progression Criterion
PIE.5.NF.A.1	2	0	2	0	0
PIE.5.NF.A.2	2	1	2	0	2
PIE.5.NF.A.3	0	0	0	0	0
PIE.5.NF.B.4	1	1	1	0	2
PIE.5.NF.B.5a	2	2	2	0	2
PIE.5.NF.B.5b	0	0	0	1	0
PIE.5.NF.B.5c	0	0	2	2	1
PIE.5.NF.B.5d	2	2	1	1	1
PIE.5.NF.B.6	2	2	0	1	2
PIE.5.NF.B.7a	2	1	3	0	3
PIE.5.NF.B.7b	1	0	1	0	1
PIE.5.NF.B.7c	2	2	2	0	2
PIE.5.NF.B.8a	2	0	1	0	2
PIE.5.NF.B.8b	1	0	1	0	1
PIE.5.RA.A.1a	2	0	0	0	0
PIE.5.RA.A.1b	0	0	2	1	2
PIE.5.RA.A.1c	1	1	1	0	2
PIE.5.RA.A.1d	0	0	2	0	2

PIE.5.RA.A.2	0	0	0	1	0
PIE.5.RA.C.5	0	0	2	0	0
PIE.5.GM.A.2	1	0	2	0	1
PIE.5.GM.B.4a	0	0	2	0	0
PIE.5.GM.B.4b	1	0	2	0	1
PIE.5.GM.C.6a	0	0	3	0	1
PIE.5.DS.A.2	1	0	1	0	1
Total	25	12	35	7	29

The collected panel feedback was also summarized for the content accuracy and structural coherence of the draft learning pathways in three pathway levels. Table 14 summarizes the number of the pathway levels that were considered to meet the content and structure review criteria. Across 25 draft learning pathways, 53 (70.7%), 63 (87.5%), and 43 (57.3%) of 75 pathway levels met the content review criteria for alignment, size, and clarity, respectively. Across 25 draft learning pathways, 70 (93.3%) and 51 (68.0%) of 75 pathway levels met the structure review criteria for consistency and progression, respectively.

Table 14.

Panel Feedback Summary—Counts of Pathway Levels That Met a Review Criterion by Review Criterion and Learning Pathway

Learning Pathway	Pathway Levels That Met a Content Review Criterion			Pathway Levels That Met a Structure Review Criterion	
	Alignment Criterion	Size Criterion	Clarity Criterion	Consistency Criterion	Progression Criterion
PIE.5.NF.A.1	1	3	1	3	3
PIE.5.NF.A.2	1	2	1	3	1
PIE.5.NF.A.3	3	3	3	3	3
PIE.5.NF.B.4	2	2	2	3	1
PIE.5.NF.B.5a	1	1	1	3	1
PIE.5.NF.B.5b	3	3	3	2	3
PIE.5.NF.B.5c	3	3	1	1	2
PIE.5.NF.B.5d	1	1	2	2	2
PIE.5.NF.B.6	1	1	3	2	1
PIE.5.NF.B.7a	1	2	0	3	0
PIE.5.NF.B.7b	2	3	2	3	2
PIE.5.NF.B.7c	1	1	1	3	1
PIE.5.NF.B.8a	1	3	2	3	1
PIE.5.NF.B.8b	2	3	2	3	2
PIE.5.RA.A.1a	1	3	3	3	3
PIE.5.RA.A.1b	3	3	1	3	2
PIE.5.RA.A.1c	3	2	2	3	1
PIE.5.RA.A.1d	3	3	1	3	2
PIE.5.RA.A.2	3	3	3	3	3

PIE.5.RA.C.5	3	3	2	3	3
PIE.5.GM.A.2	3	3	1	3	3
PIE.5.GM.B.4a	3	3	1	3	3
PIE.5.GM.B.4b	3	3	1	3	3
PIE.5.GM.C.6a	3	3	2	3	3
PIE.5.DS.A.2	2	3	2	3	2
Total	53	63	43	70	51

Panel feedback analysis and decision

The panel feedback analysis and pathway change identification stages of the post-expert review process produced the counts of accepted panel feedback by counting the number of pathway levels determined to be revised and refined and types of indicated changes to the pathway levels (e.g., counting the number of pathway levels coded for clarity changes). Table 15 summarizes the decisions made to either accept or reject the panel feedback on the draft learning pathways' content and structure.

Across the 75 pathway levels, 12 (16.7%) of the 72 pathway levels for which the expert panel provided panel feedback on the content of the draft learning pathways were accepted and 6 (16.7%) of the 36 pathway levels for which the expert panel provided panel feedback on the structure of the draft learning pathways were accepted. The rationales provided with panel feedback were considered in the decisions to accept or reject the panel feedback. For instance, for the draft learning pathway level statement of "Compare and order volumes by direct comparison," the expert panel flagged this learning pathway level statement for its content clarity and recommended to "Change level statement to 'order 2 or more volumes by direct comparison.' Rationale: To better match the nodes and let the teacher know they should go beyond 2." However, this panel feedback was rejected based on the reasons that the recommended inclusion is not necessary since ordering requires more than two volumes and this content change may add ambiguity to the learning pathway level statement's "Compare" portion.

Table 15.

Panel Feedback Decision Summary—Counts of Collected, Accepted, and Rejected Panel Feedback on Pathway Level Content and Structure by Learning Pathway

Learning Pathway	Collected Panel Feedback on Pathway Level Content	Decisions That Were Made to Panel Feedback on Pathway Level Content		Collected Panel Feedback on Pathway Level Structure	Decisions That Were Made to Panel Feedback on Pathway Level Structure	
		Accepted Panel Feedback	Rejected Panel Feedback		Accepted Panel Feedback	Rejected Panel Feedback
PIE.5.NF.A.1	4	0	4	0	-	-
PIE.5.NF.A.2	5	0	5	2	0	2
PIE.5.NF.A.3	0	-	-	0	-	-

PIE.5.NF.B.4	3	0	3	2	0	2
PIE.5.NF.B.5a	6	0	6	2	0	2
PIE.5.NF.B.5b	0	-	-	1	0	1
PIE.5.NF.B.5c	2	0	2	3	0	3
PIE.5.NF.B.5d	5	0	5	2	0	2
PIE.5.NF.B.6	4	0	4	3	0	3
PIE.5.NF.B.7a	6	2	4	3	1	2
PIE.5.NF.B.7b	2	0	2	1	0	1
PIE.5.NF.B.7c	6	6	0	2	2	0
PIE.5.NF.B.8a	3	3	0	2	2	0
PIE.5.NF.B.8b	2	0	2	1	0	1
PIE.5.RA.A.1a	2	1	1	0	-	-
PIE.5.RA.A.1b	2	0	2	3	0	3
PIE.5.RA.A.1c	3	0	3	2	0	2
PIE.5.RA.A.1d	2	0	2	2	0	2
PIE.5.RA.A.2	0	-	-	1	0	1
PIE.5.RA.C.5	2	0	2	0	-	-
PIE.5.GM.A.2	3	0	3	1	1	0
PIE.5.GM.B.4a	2	0	2	0	-	-
PIE.5.GM.B.4b	3	0	3	1	0	1
PIE.5.GM.C.6a	3	0	3	1	0	1
PIE.5.DS.A.2	2	0	2	1	0	1
Total	72	12	60	36	6	30

The accepted panel feedback was also summarized for the pathway level content and structure changes indicated by change codes that represented the type of change. Table 16 summarizes the number of pathway level content and structure changes decided to make to the draft learning pathways. In total, across the 75 pathway levels from 25 draft learning pathways, 19 content changes and 13 structure changes were indicated based on the accepted panel feedback. Fourteen (73.7%) of the 19 pathway level content changes involve alignment improvements to ensure a clear link to the content standards' learning targets. Five (26.3%) of the 19 content changes were to improve clarity and consistency of the language addressing the KSUs of interest. One (7.7%), six (46.15%), and six (46.15%) of the 13 pathway level structure changes involve the transfer of node(s) from one pathway level to another, inclusion of node(s) to pathway levels, and exclusion of node(s) from pathway levels, respectively, for the sake of vertical articulation within a learning pathway.

Table 16.

Pathway Level Change Summary—Counts of Changes That Were Decided to Make to Pathway Level Content and Structure by Change Type and Learning Pathway

Learning Pathway	Changes That Were Decided to Make to Pathway Level Content	Changes That Were Decided to Make to Pathway Level Structure
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	Alignment Change	Size Change	Clarity Change	Transfer	Inclusion	Exclusion
PIE.5.NF.A.1	0	0	0	0	0	0
PIE.5.NF.A.2	0	0	0	0	0	0
PIE.5.NF.A.3	0	0	0	0	0	0
PIE.5.NF.B.4	0	0	1	0	0	0
PIE.5.NF.B.5a	0	0	0	0	0	0
PIE.5.NF.B.5b	0	0	0	0	0	0
PIE.5.NF.B.5c	0	0	0	0	0	0
PIE.5.NF.B.5d	0	0	0	0	0	0
PIE.5.NF.B.6	0	0	0	0	0	0
PIE.5.NF.B.7a	2	0	0	0	1	1
PIE.5.NF.B.7b	0	0	1	0	0	0
PIE.5.NF.B.7c	4	0	0	0	2	2
PIE.5.NF.B.8a	4	0	0	1	1	1
PIE.5.NF.B.8b	0	0	0	0	0	0
PIE.5.RA.A.1a	2	0	0	0	1	1
PIE.5.RA.A.1b	0	0	0	0	0	0
PIE.5.RA.A.1c	0	0	0	0	0	0
PIE.5.RA.A.1d	0	0	0	0	0	0
PIE.5.RA.A.2	2	0	0	0	1	1
PIE.5.RA.C.5	0	0	0	0	0	0
PIE.5.GM.A.2	0	0	3	0	0	0
PIE.5.GM.B.4a	0	0	0	0	0	0
PIE.5.GM.B.4b	0	0	0	0	0	0
PIE.5.GM.C.6a	0	0	0	0	0	0
PIE.5.DS.A.2	0	0	0	0	0	0
Total	14	0	5	1	6	6

Finalized Learning Pathway Approval

Following the post-expert review process, the 19 content changes and 13 structure changes, as summarized in Table 16, were made to the content and structure of the draft learning pathways with three pathway levels, resulting in the finalized learning pathways.³ The Missouri DESE approved all 25 of the finalized learning pathways with three pathway levels as being ready for PIE assessment development in August 2023.

³ In June 2023, the learning pathway development process, which included expert review, was presented to the Project Advisory Committee (PAC). The PAC supported the use of the pathway levels in assessment development. The PAC also provided suggestions regarding potential teacher resources that could be developed to support teachers' use and interpretation of the learning pathways.

Conclusion

To achieve the project objective, the PIE project constructed 25 learning pathways as student learning paths to the selected Missouri grade 5 mathematics content standards from the Priority Standards for Leveraging Learning in Mathematics (Missouri DESE, 2021), based on the student learning and development literature. Then, 75 pathway levels were indicated from the learning pathways (three vertical levels for each learning pathway), following the established learning map development process that includes expert review (Bechard et al., 2021; DLM Consortium, 2016; Swinburne Romine et al., 2018).

The expert review resulted in producing extensive panel feedback on the content and structure of the 25 draft learning pathways in three pathway levels, as well as evidence of the learning pathway development process. The PIE project refined the pathway levels of the draft learning pathways using the collected panel feedback from the expert review. The resulting learning pathways serve as the underlying structure of the aligned assessments and reports for the PIE assessment system.

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Practice Activity Learning Pathway Map Document

Learning Pathway Map

Practice Activity

PIE.5.GM.D.8

Mathematics

Geometry and Measurement (GM)

Grade 5

This document provides (a) the target grade-level content standard; (b) three levels of a learning pathway aligned with the learning target; (c) the knowledge, skills, and understandings associated with each level; and (d) a map view of the full learning pathway.

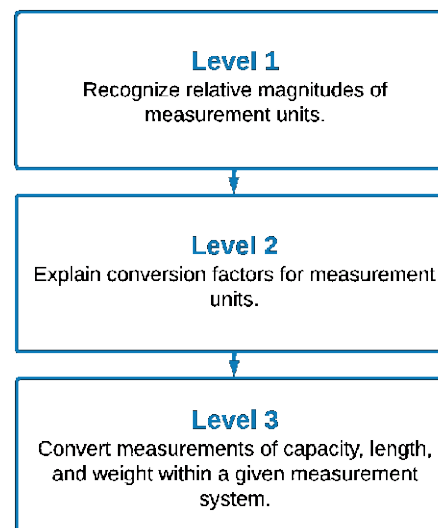
Learning Target

5.GM.D. Solve problems involving measurement and conversions within a measurement system.

8. Convert measurements of capacity, length, and weight within a given measurement system.

Learning Pathway in Three Levels

The learning pathway presents three vertical levels that consist of knowledge, skills, and understandings that build toward and meet the learning target. **Level 1** represents emerging concepts and skills related to the learning target. **Level 2** represents concepts and skills approaching the learning target. **Level 3** represents the learning target and aligns with the grade-level content standard.



PIE.5.GM.D.8

Page 1 of 3

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Knowledge, Skills, and Understandings in Three Levels

Level 1

PIE-M-10: Recognize metric units for liquid volume or capacity, length, and mass or weight that are smaller or larger than a given metric unit. *For example, when presented with a metric unit (e.g., centimeter), the student identifies metric units that are smaller (e.g., millimeter) or larger (e.g., meter).*

PIE-M-11: Recognize English units for liquid volume or capacity, length, and mass or weight that are smaller or larger than a given English unit. *For example, when presented with an English unit (e.g., pint), the student identifies English units that are smaller (e.g., cup) or larger (e.g., quart).*

Level 2

PIE-M-12: Make known your understanding through any accessible mode of expression how to convert among metric units for liquid volume or capacity, length, and mass or weight measurements. *For example, when asked how to convert between metric units (e.g., kilograms and grams), the student communicates the conversion factor (e.g., 1 kilogram = 1,000 grams).*

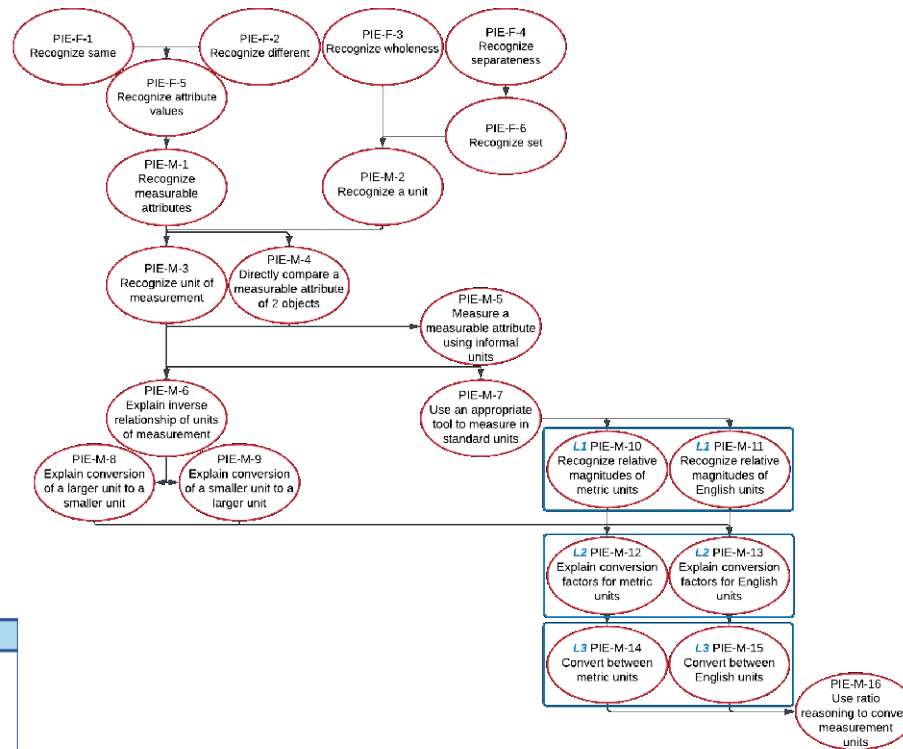
PIE-M-13: Make known your understanding through any accessible mode of expression how to convert among English units for liquid volume or capacity, length, and mass or weight measurements. *For example, when asked how to convert between English units (e.g., feet and inches), the student communicates the conversion factor (e.g., 1 foot = 12 inches).*

Level 3

PIE-M-14: Multiply or divide by the appropriate conversion factor to determine the equivalent liquid volume or capacity, length, and mass or weight measurement in a neighboring metric unit. *For example, when presented with a measurement in metric units (e.g., 8.4 centimeters), the student converts the measurement into a neighboring metric unit (e.g., 84 millimeters).*

PIE-M-15: Multiply or divide by the appropriate conversion factor to determine the equivalent liquid volume or capacity, length, and mass or weight measurement in a neighboring English unit. *For example, when presented with a measurement in English units (e.g., 4 pounds), the student converts the measurement into a neighboring English unit (e.g., 64 ounces).*

PIE.5.GM.D.8 Learning Pathway Map View



Map Key	
L1	Level 1
L2	Level 2
L3	Level 3
Boxes indicate tested nodes	

PIE.5.GM.D.8

Page 3 of 3